

ZipLex

Verified Linear-Time Invertible Lexer

Samuel Chassot, Viktor Kunčák

27.07.2026 - CAV26

Sorting Functions

Refactor Tool Example

```
def abs(x: Int): Int =  
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    x  
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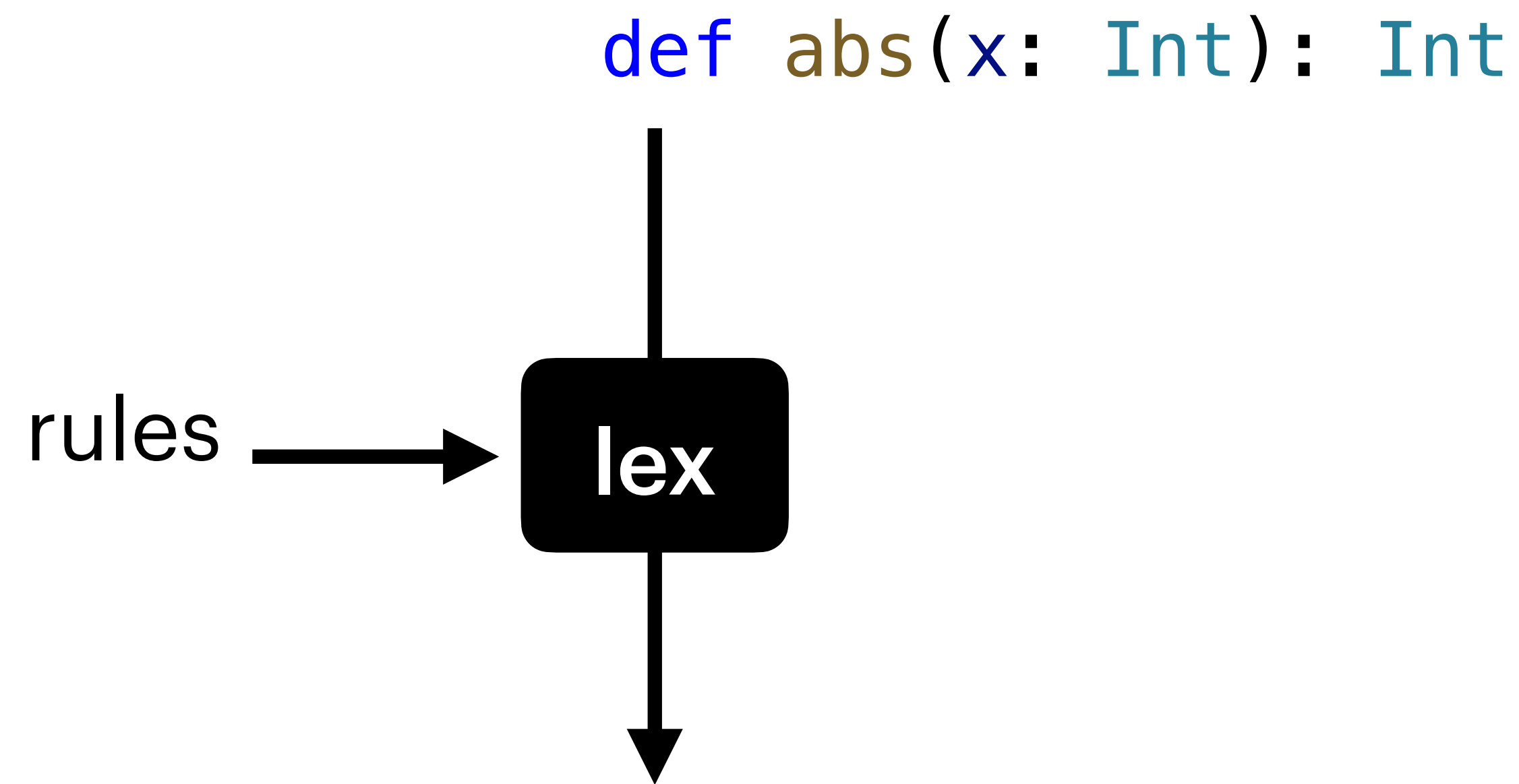
Lexical Analysis

Ziplex Interface

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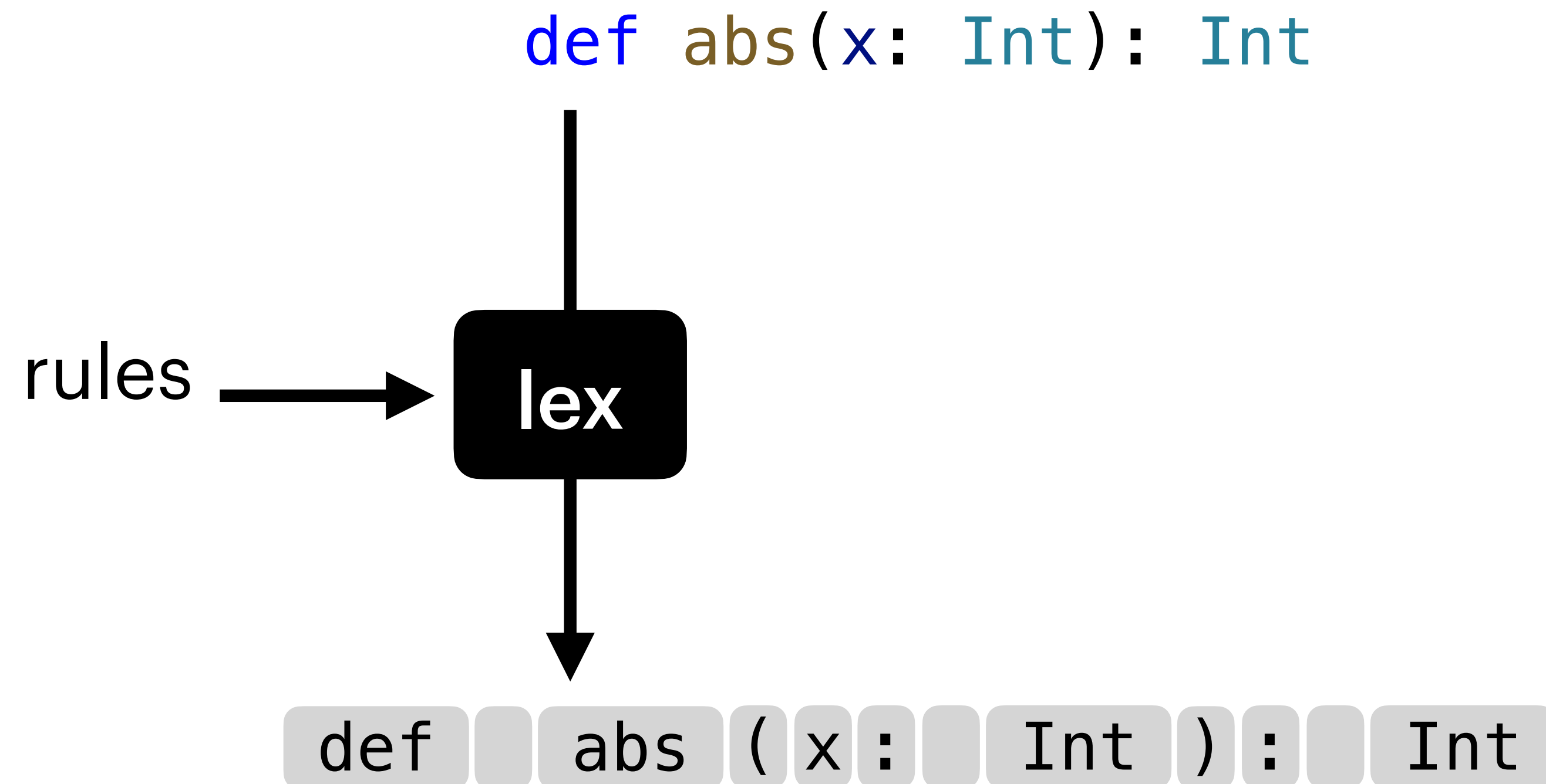
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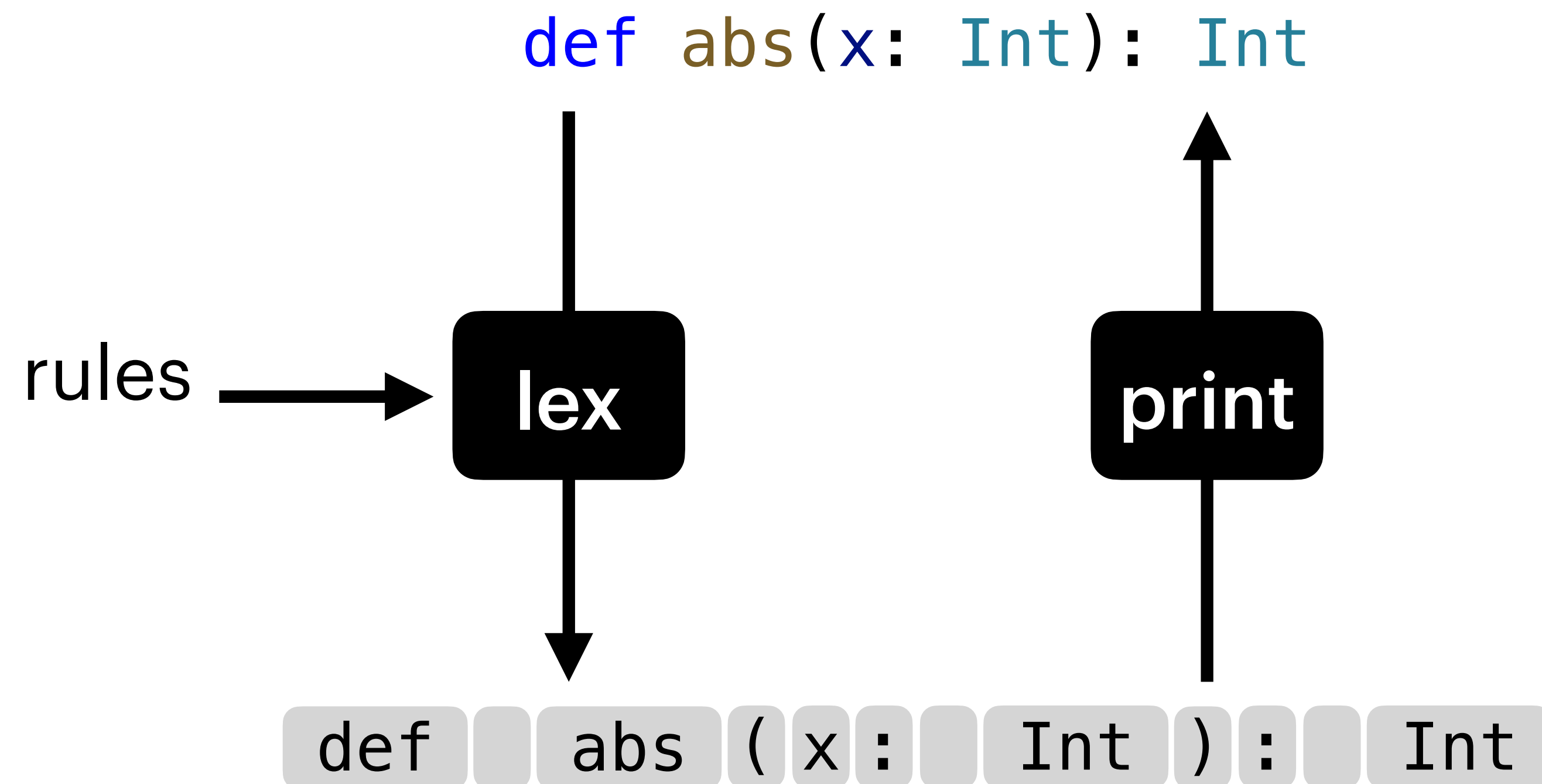
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Lexical Analysis

Ziplex Interface



ZipLex Framework

Scala project, verified with Stainless



ZipLex

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ZipLex

Regular expression
engine

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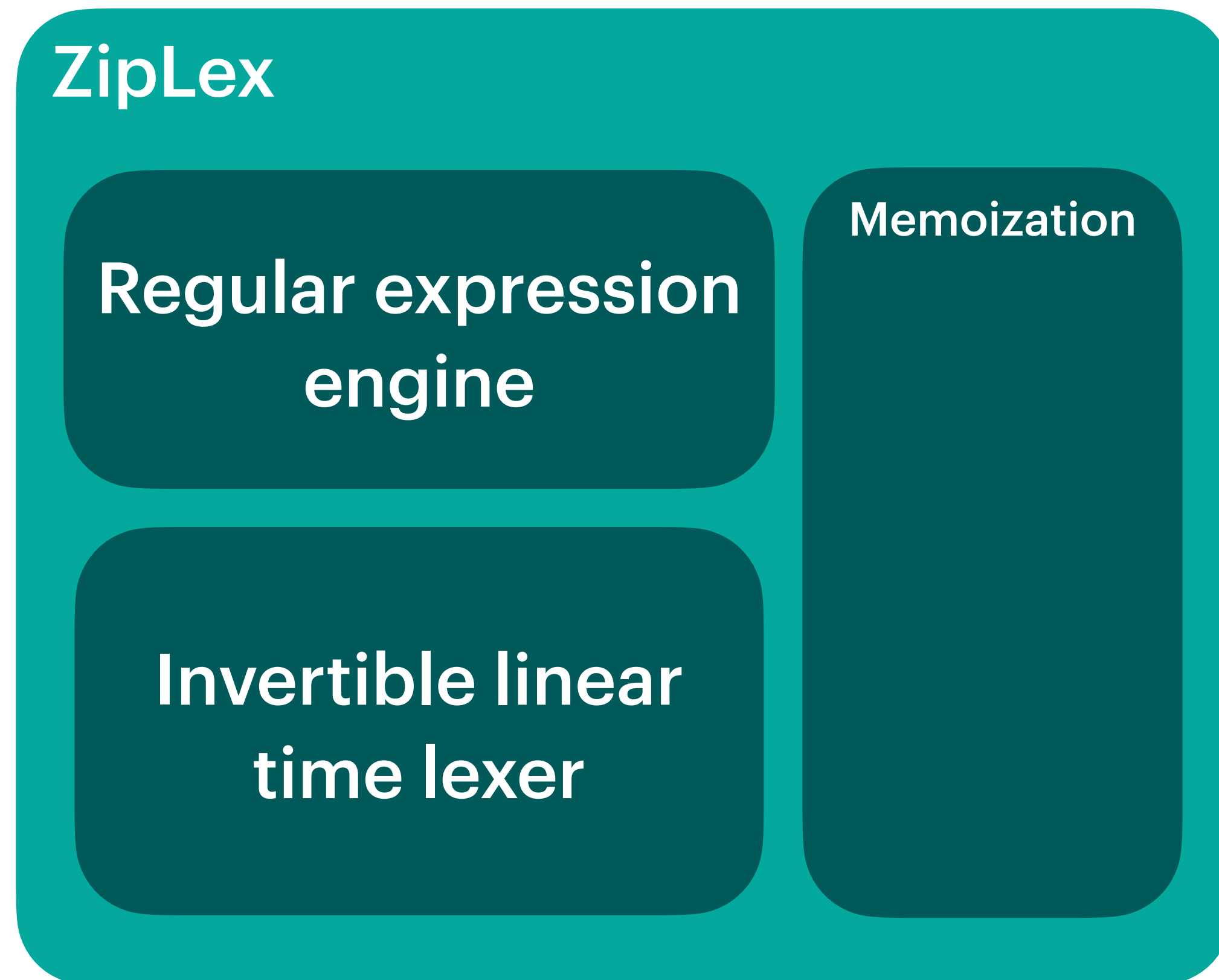
ZipLex

Regular expression
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Invertible linear
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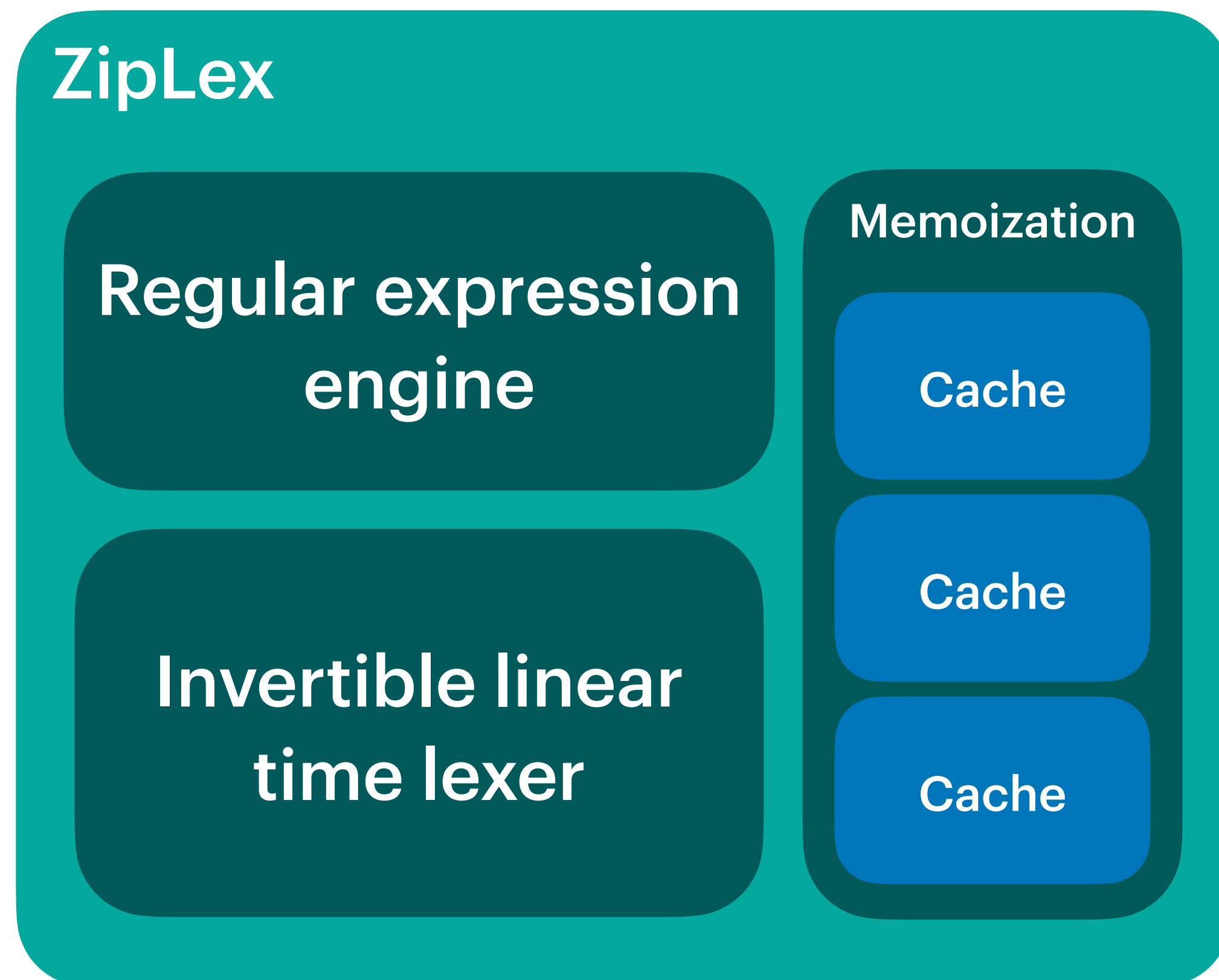
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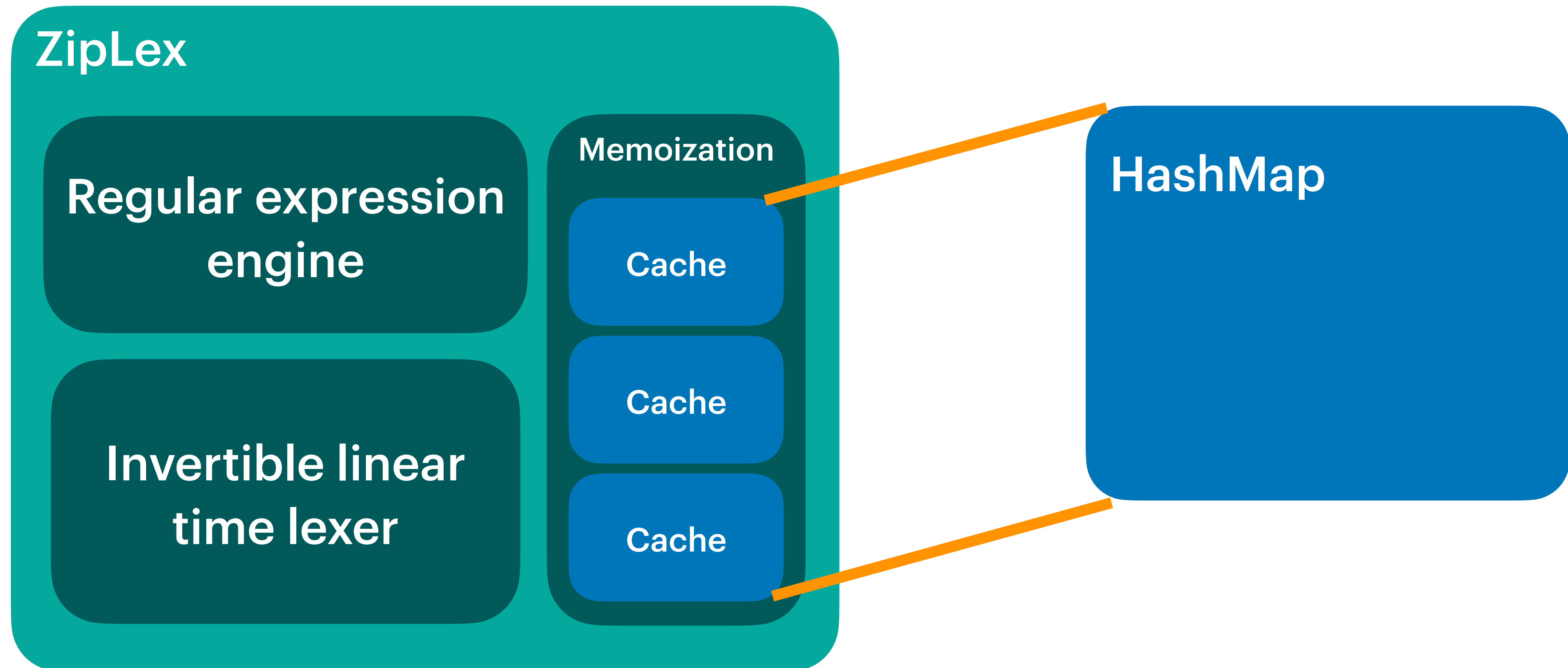
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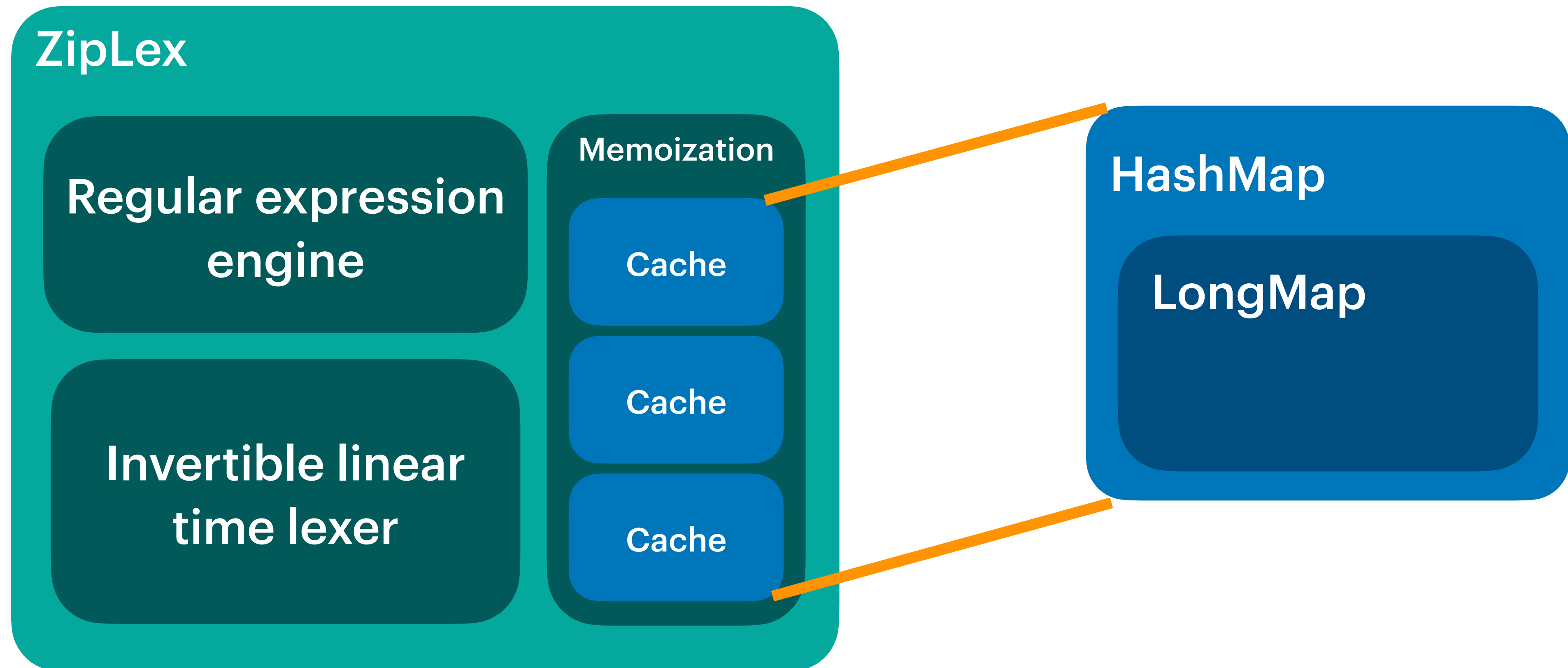
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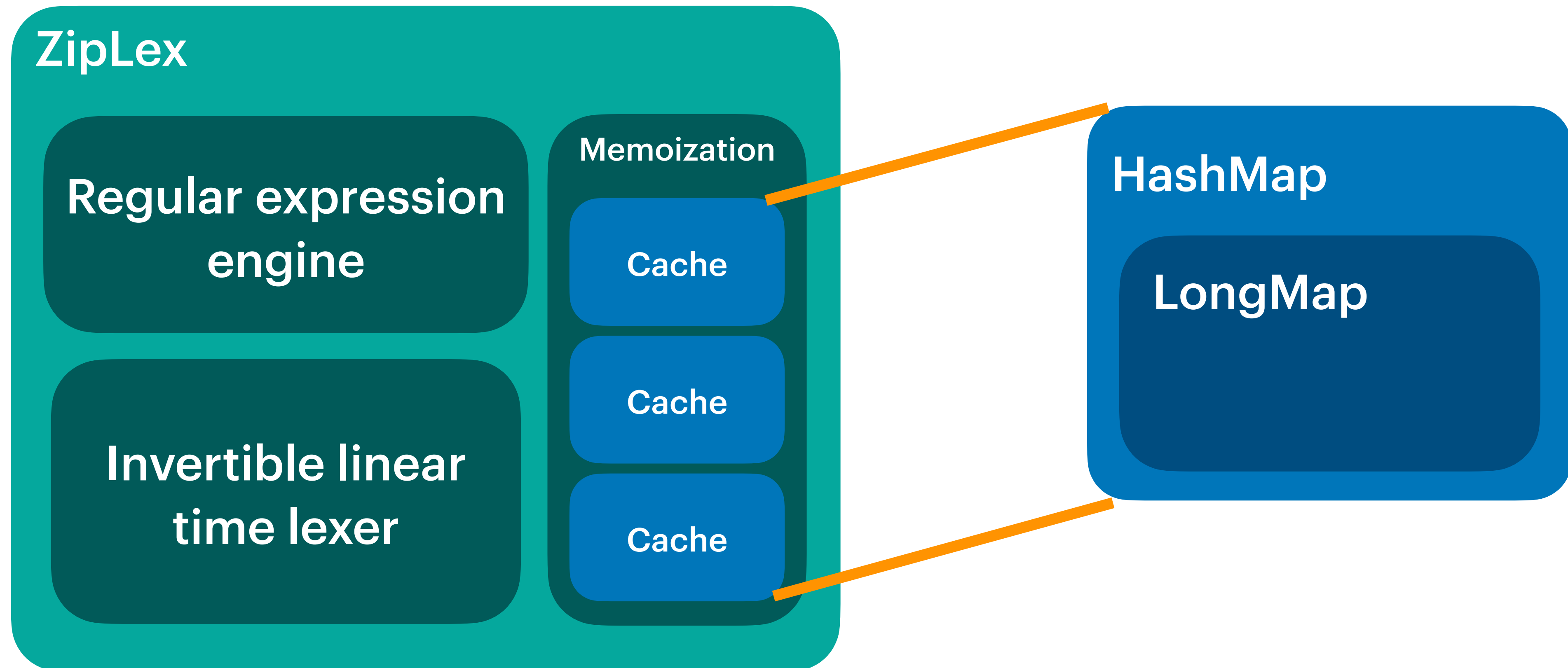
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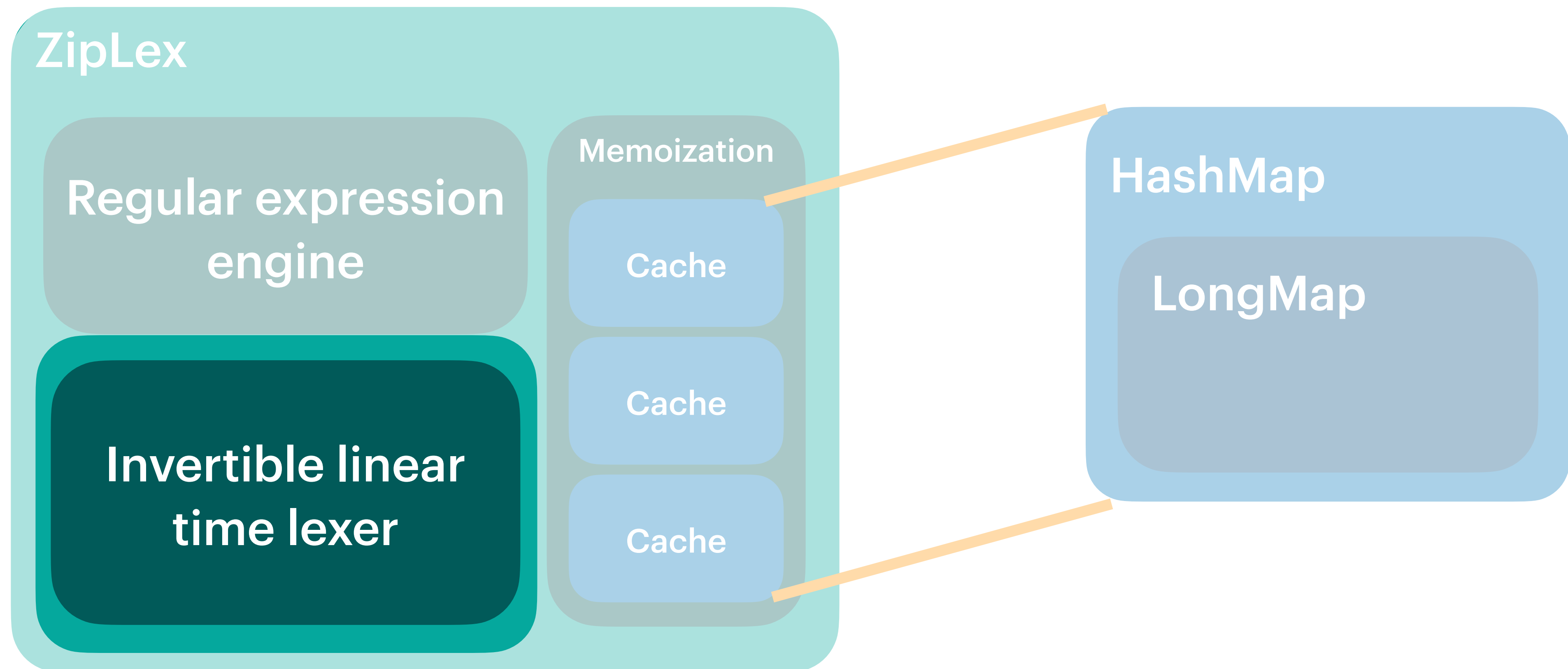
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Composition pattern for efficient programs verification

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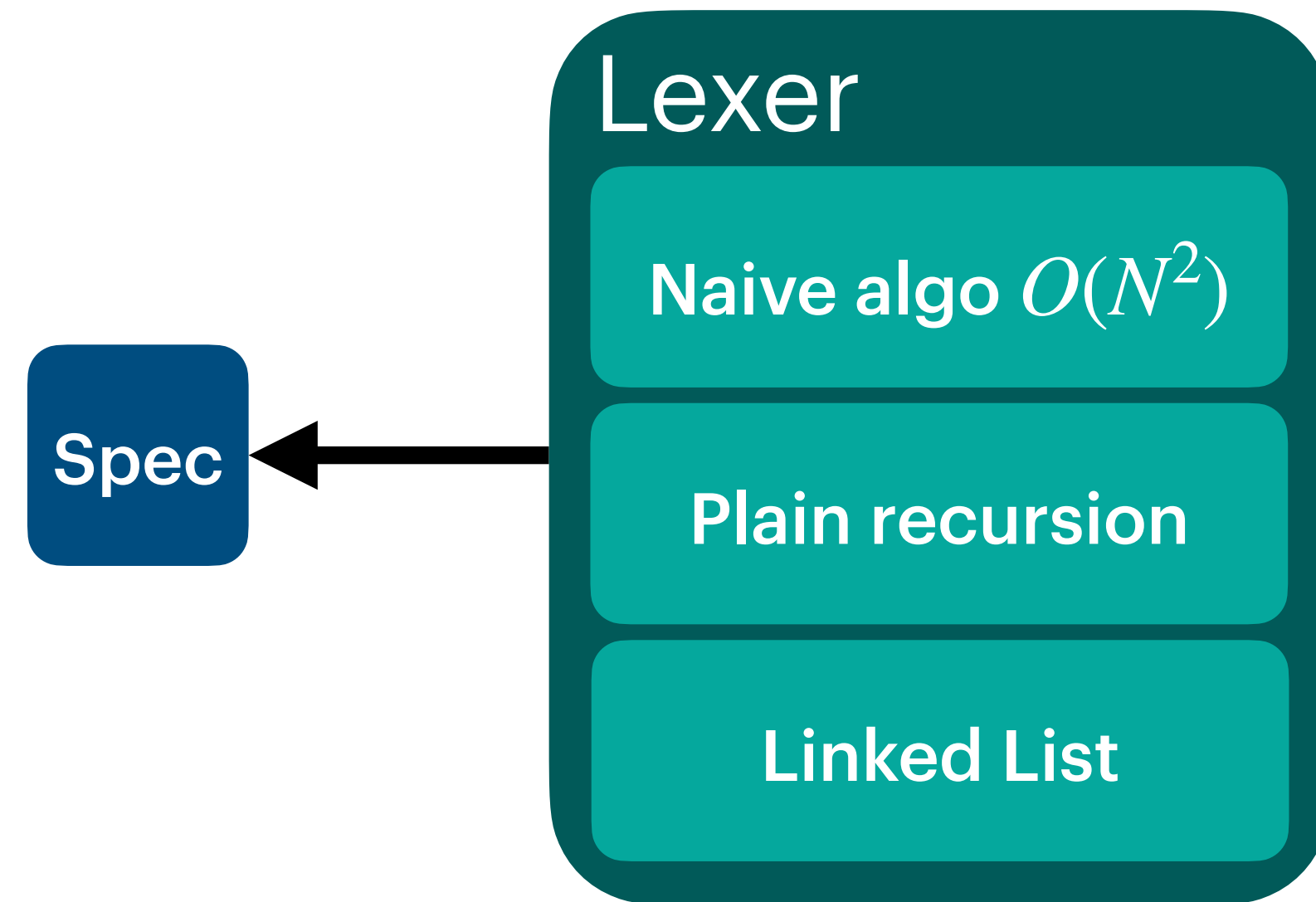
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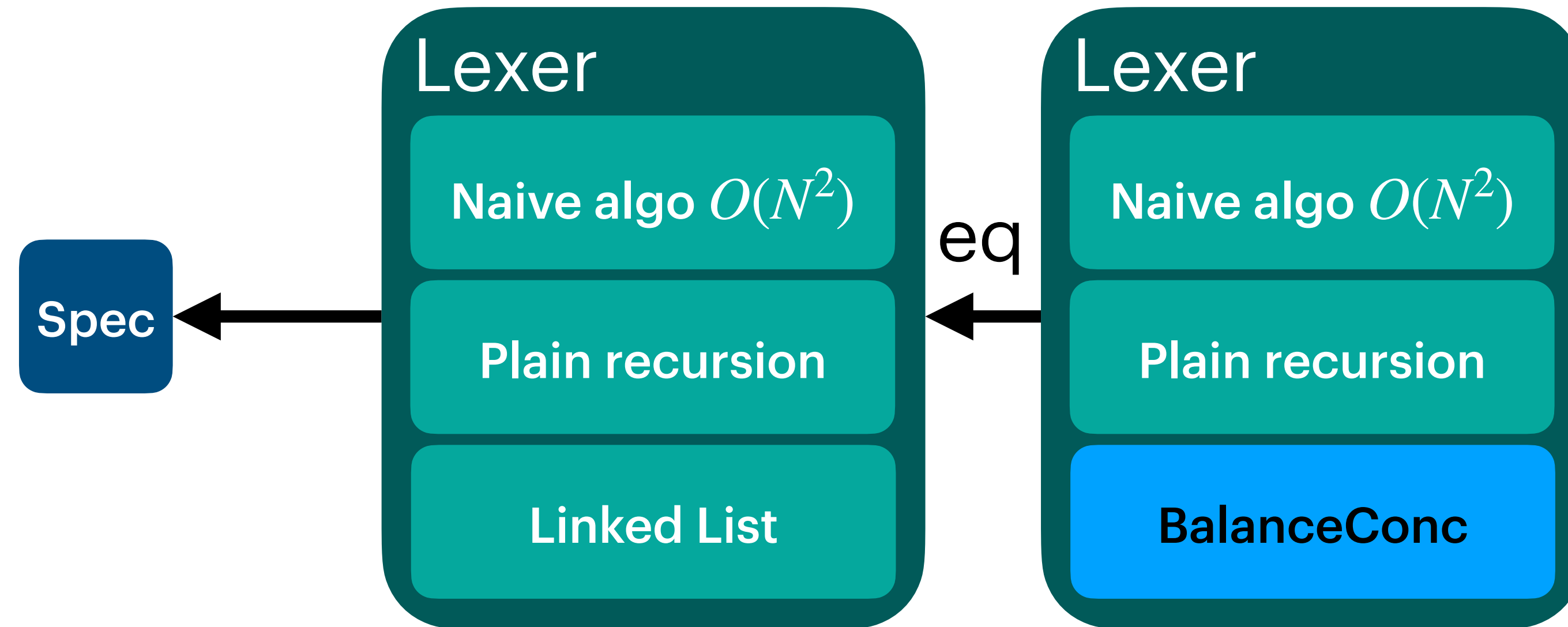
Composition pattern for efficient programs verification

Implementation and verification methodology

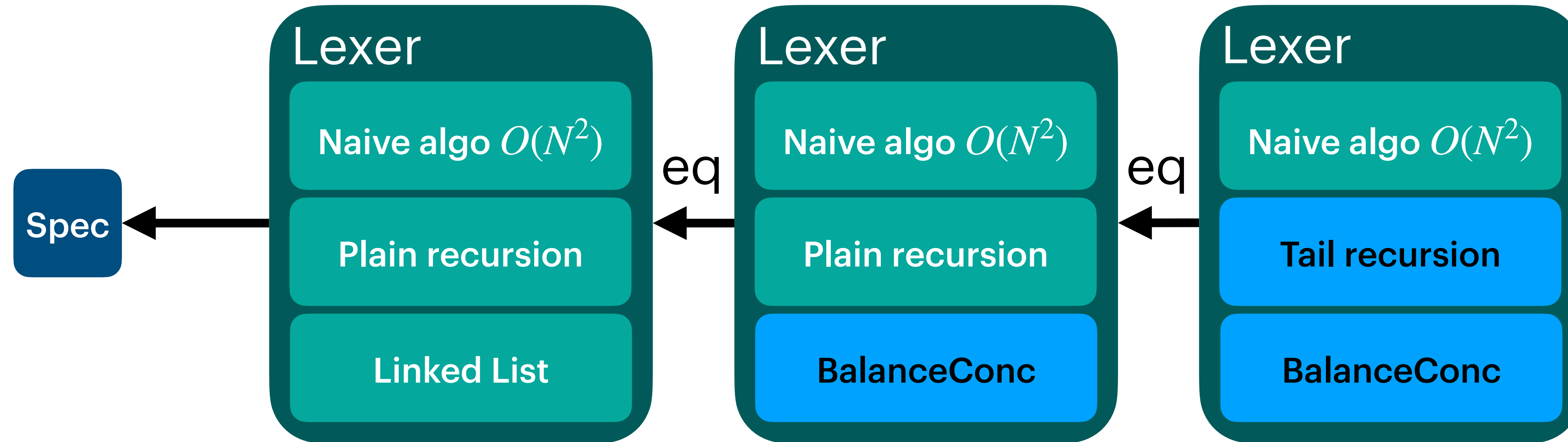
Implementation and verification methodology



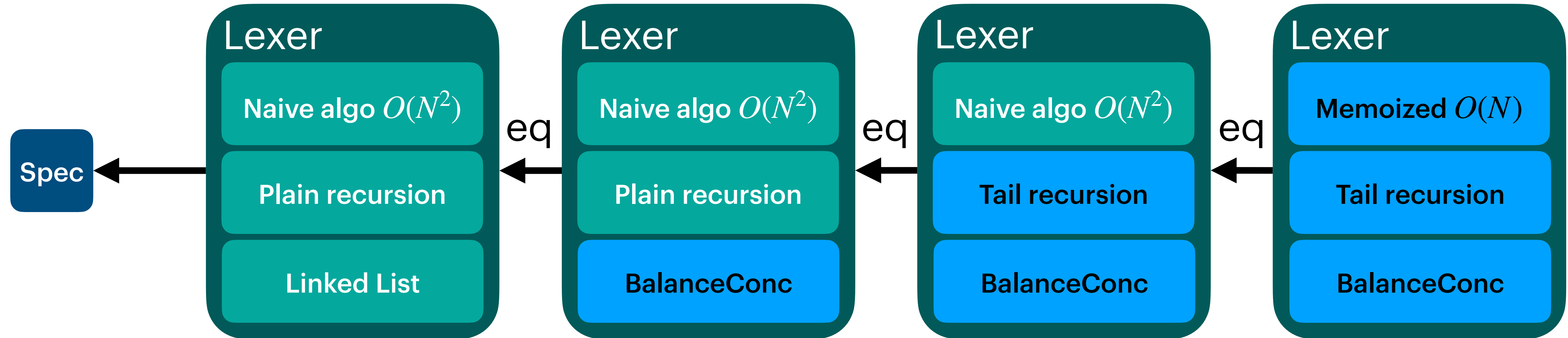
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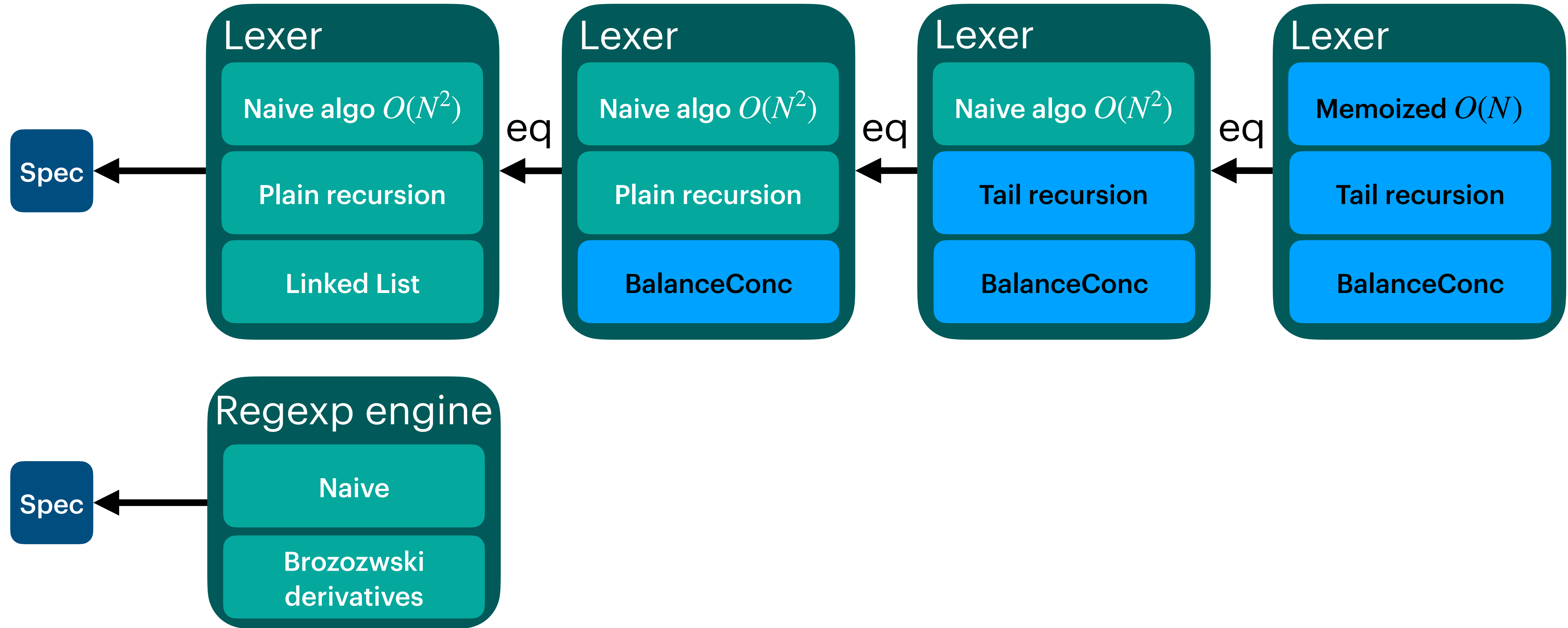
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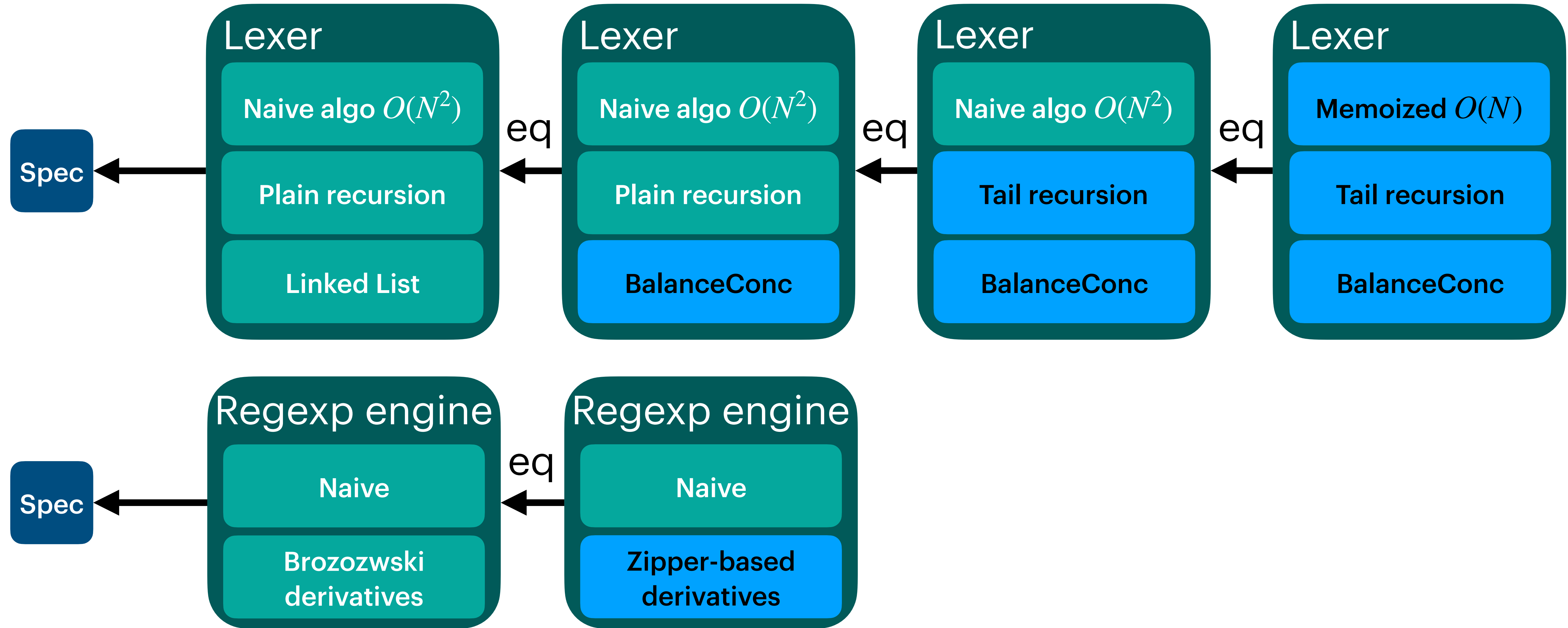
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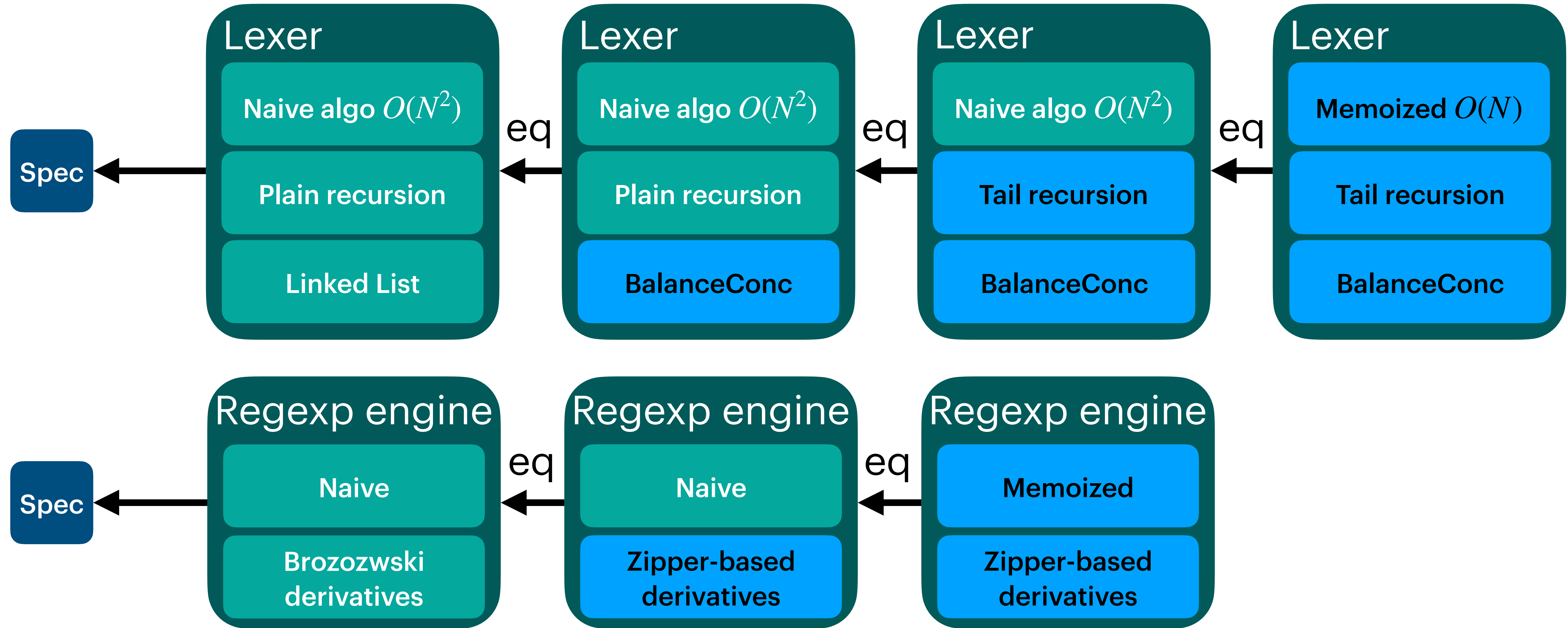
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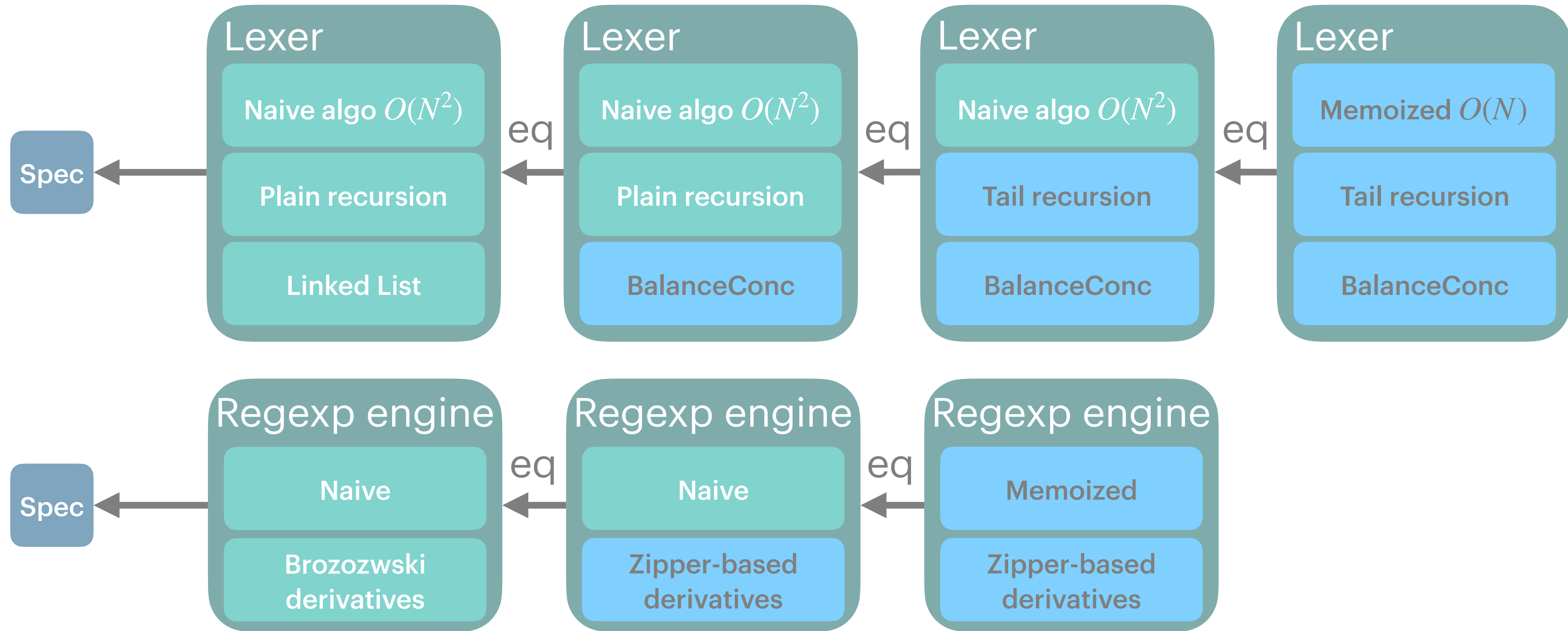
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Implementation and verification methodology



Iterative approach to efficiently verify efficient programs

Specification

Introduction

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Longest match

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Longest possible prefix for each token

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Invertible lexer if both

$$print(lex(s)) = s \quad \text{and} \quad lex(print(ts)) = ts$$

Lexing Invertibility

2. *lex(print(ts)) = ts*

Tokens can ***collapse***

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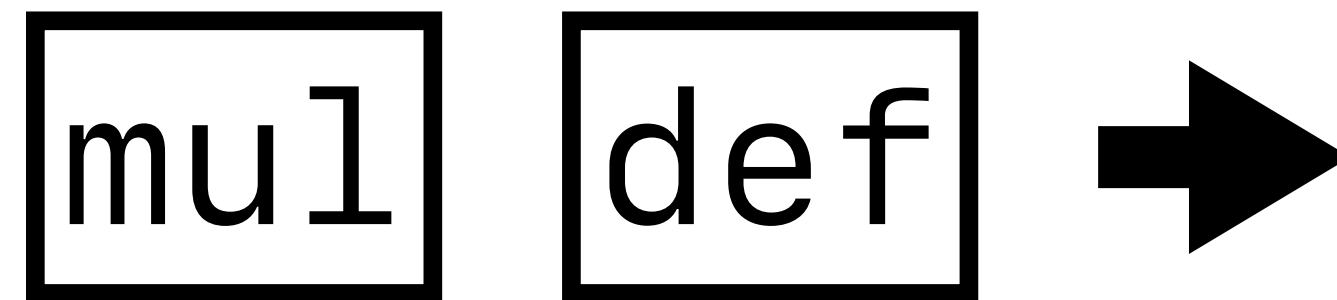
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mul def

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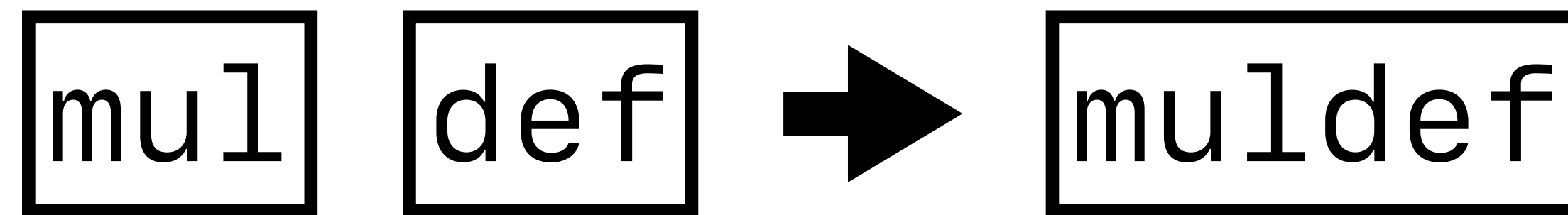
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Lexing Invertibility

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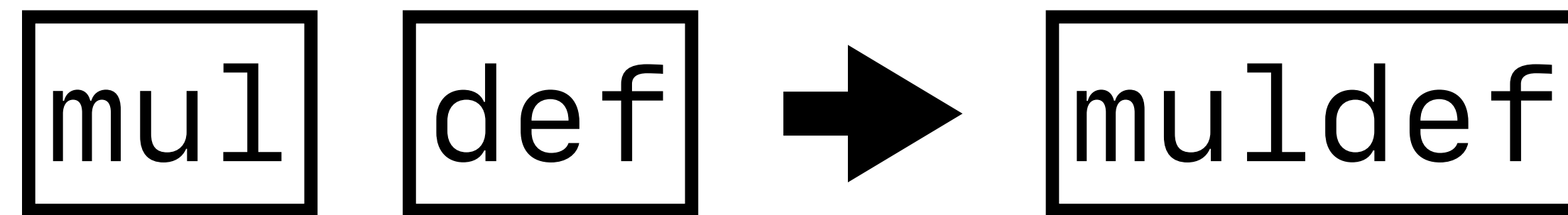
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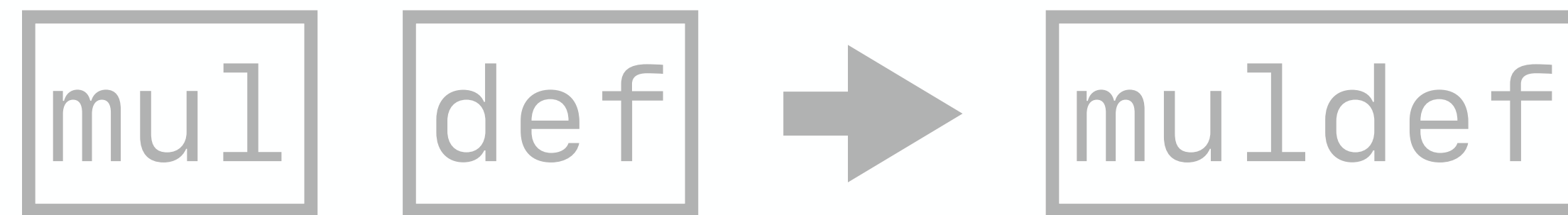


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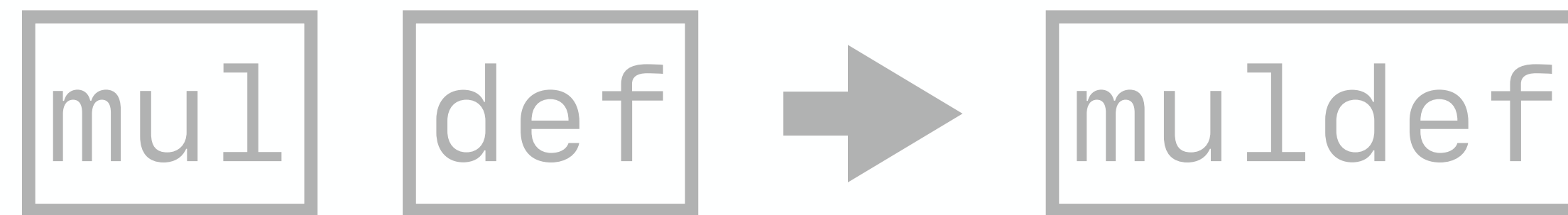
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We define the notion of **separability**

$\text{sep}(ts) \implies \text{lex}(\text{print}(ts)) = ts$

Token Separability

Predicate definition

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Look only at adjacent tokens

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R-Paths

Introduction

Given a binary relation R , a sequence $s = s_1, s_2, \dots, s_n$ is an *R-Path* if the relation holds for each consecutive pair

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s_1

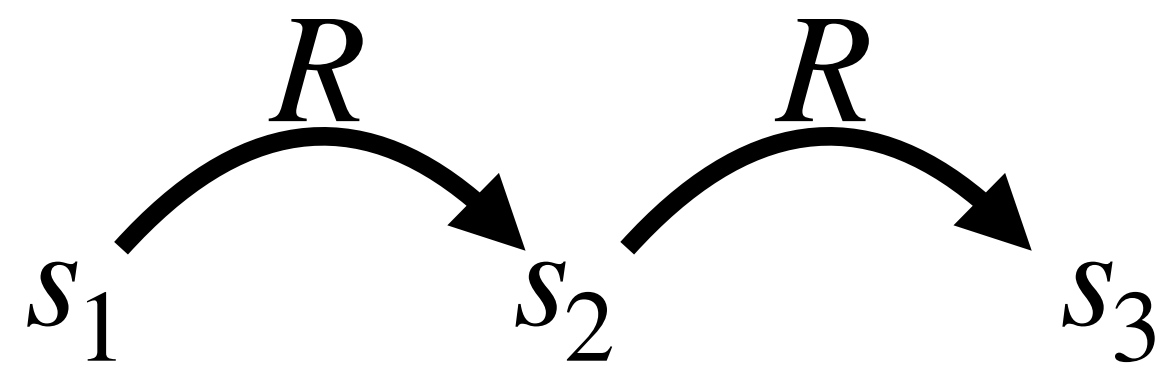
s_2

s_3

R-Paths

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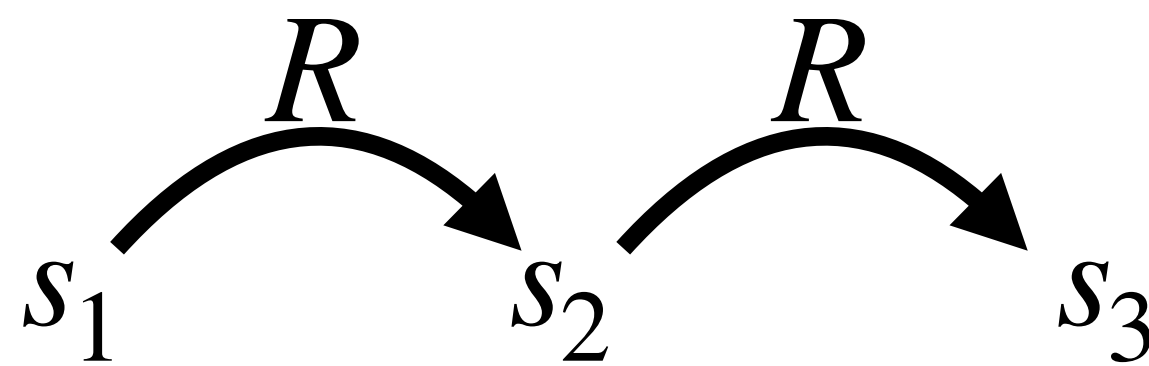
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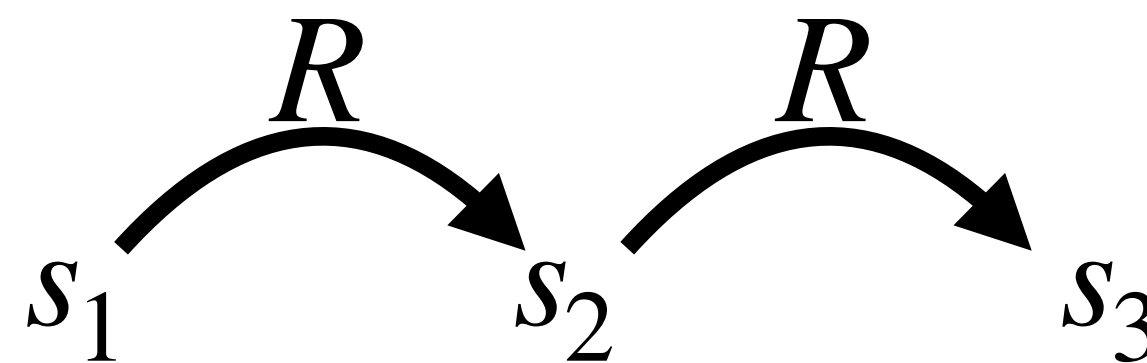
Formally, *R-Path* predicate

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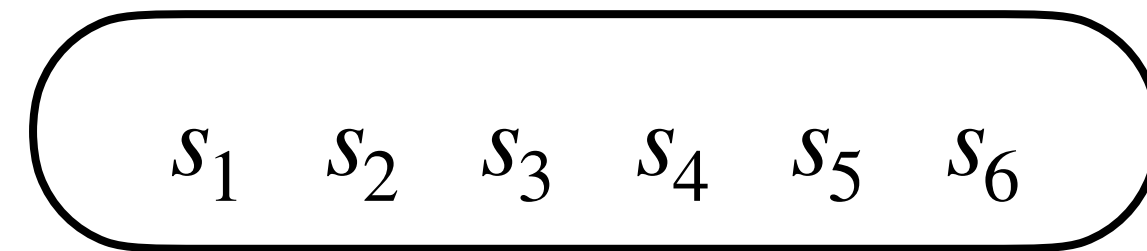
Predicates over sequences requiring only local computations

R-Path

Efficient operations with statically checked invariant

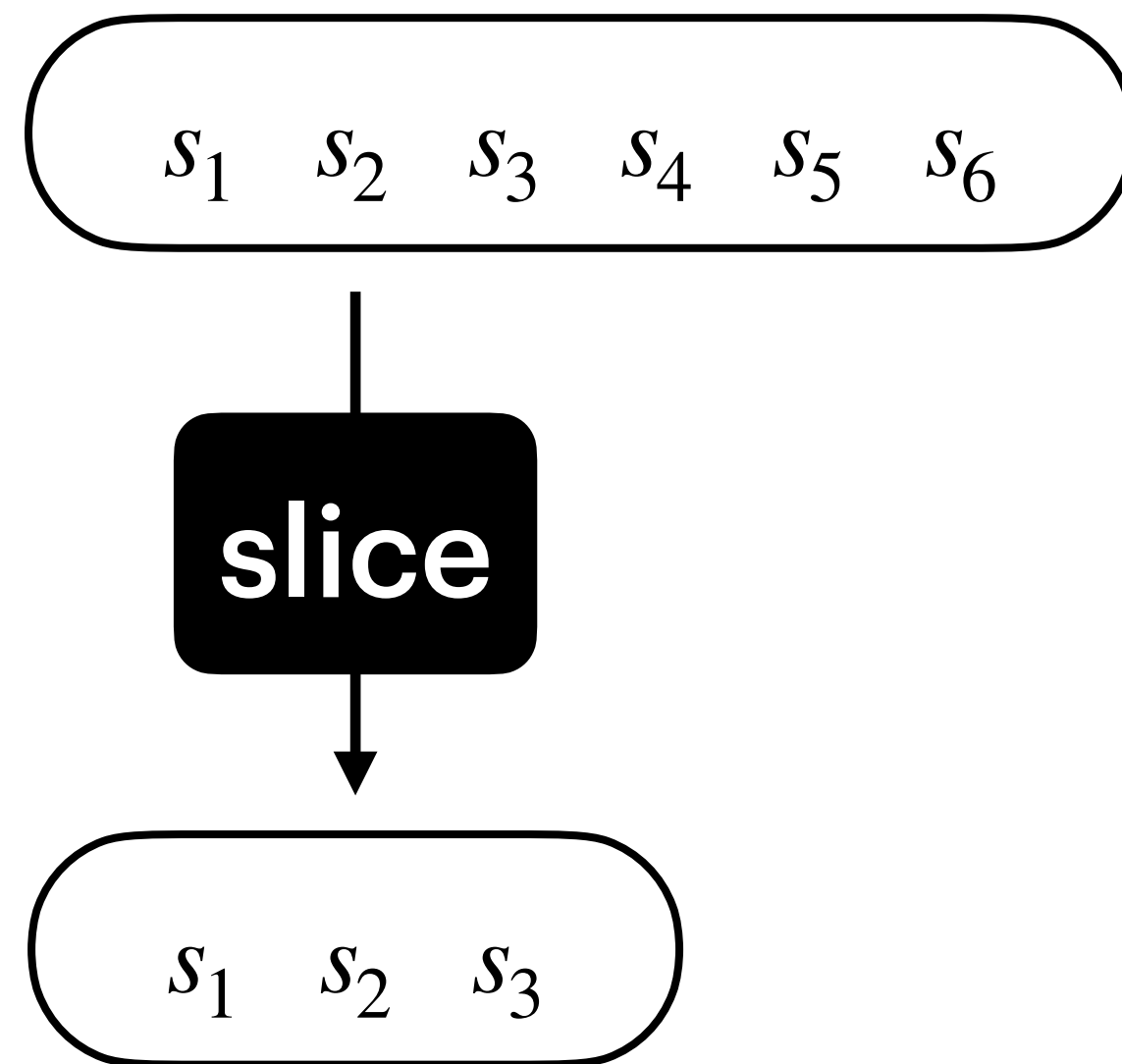
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R-Path

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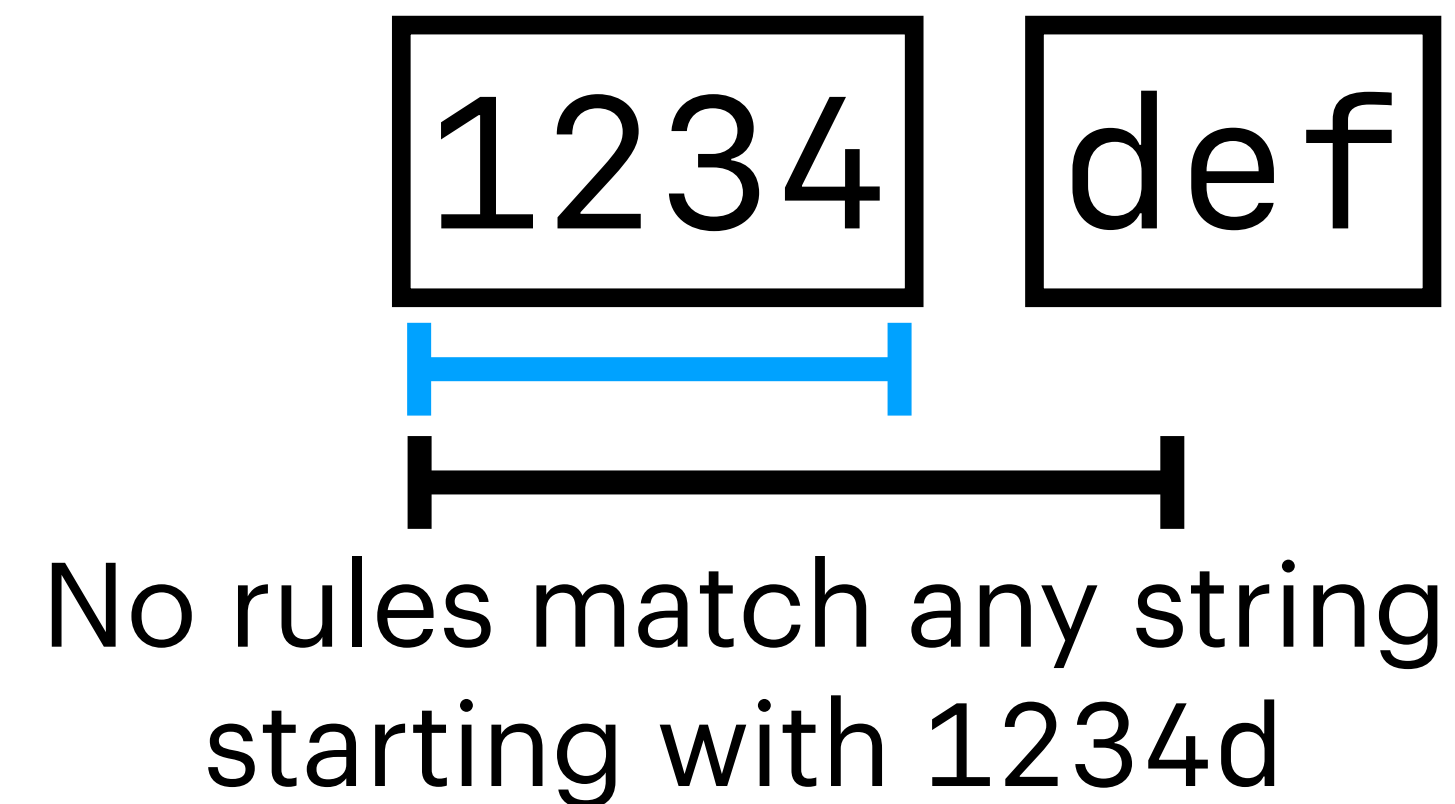

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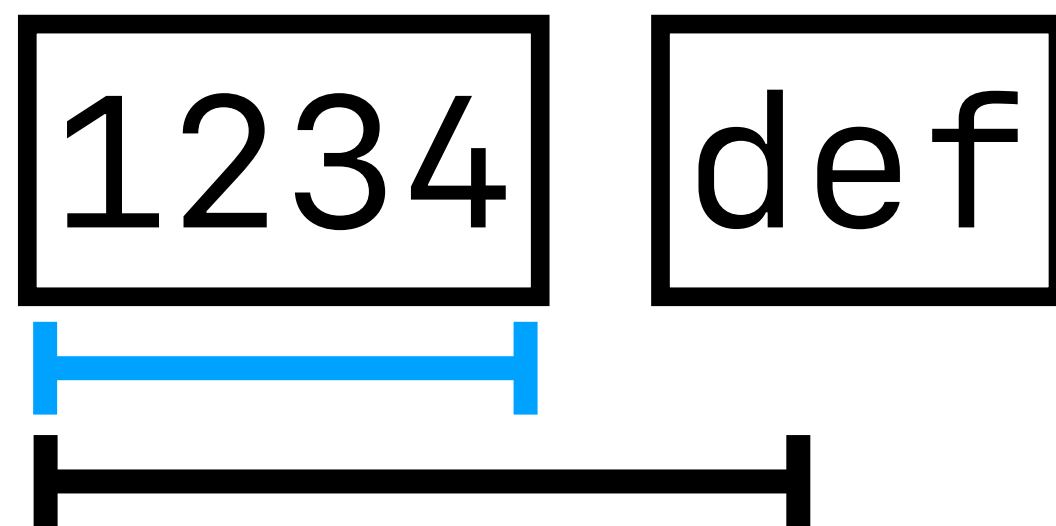


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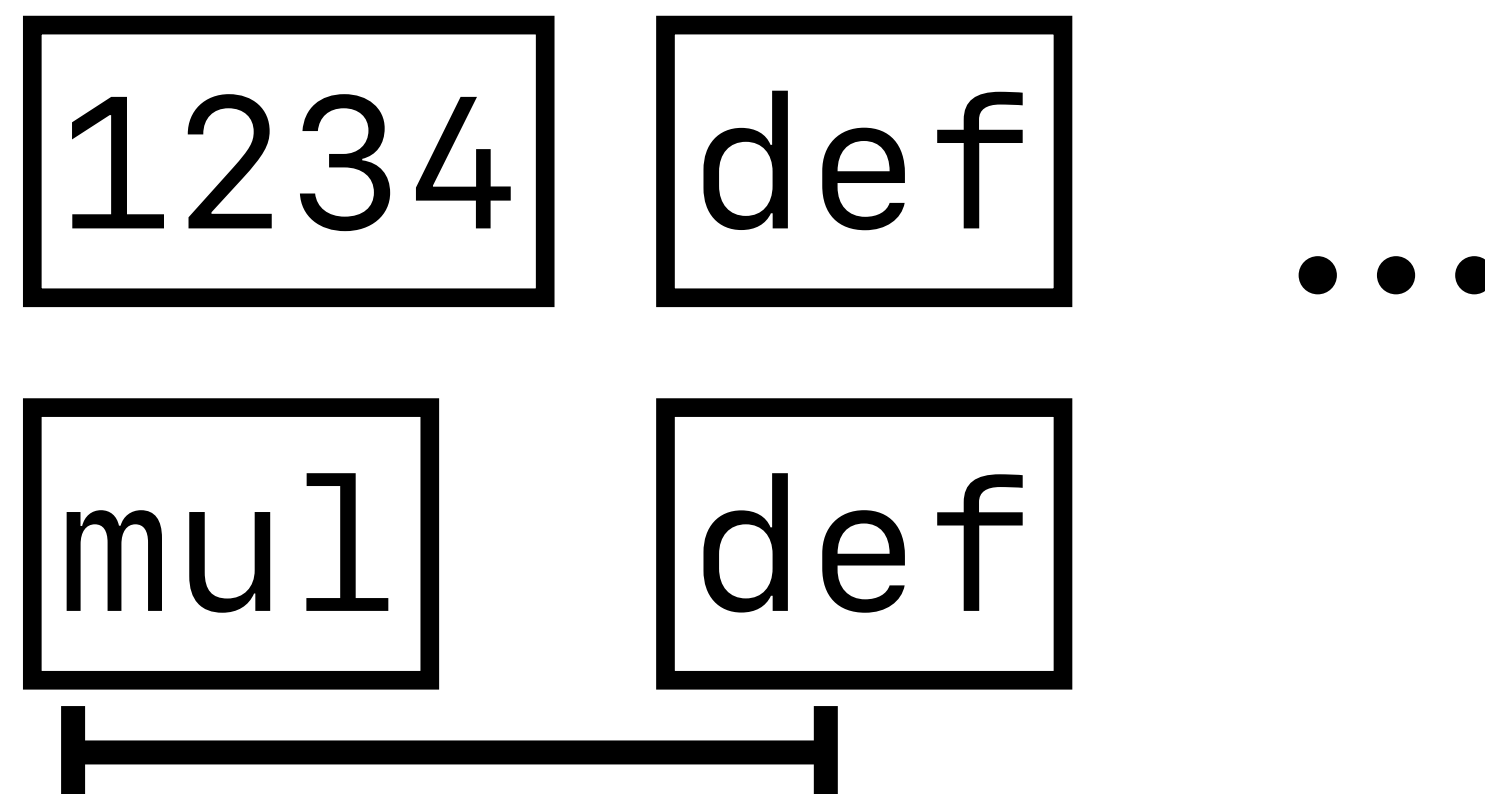


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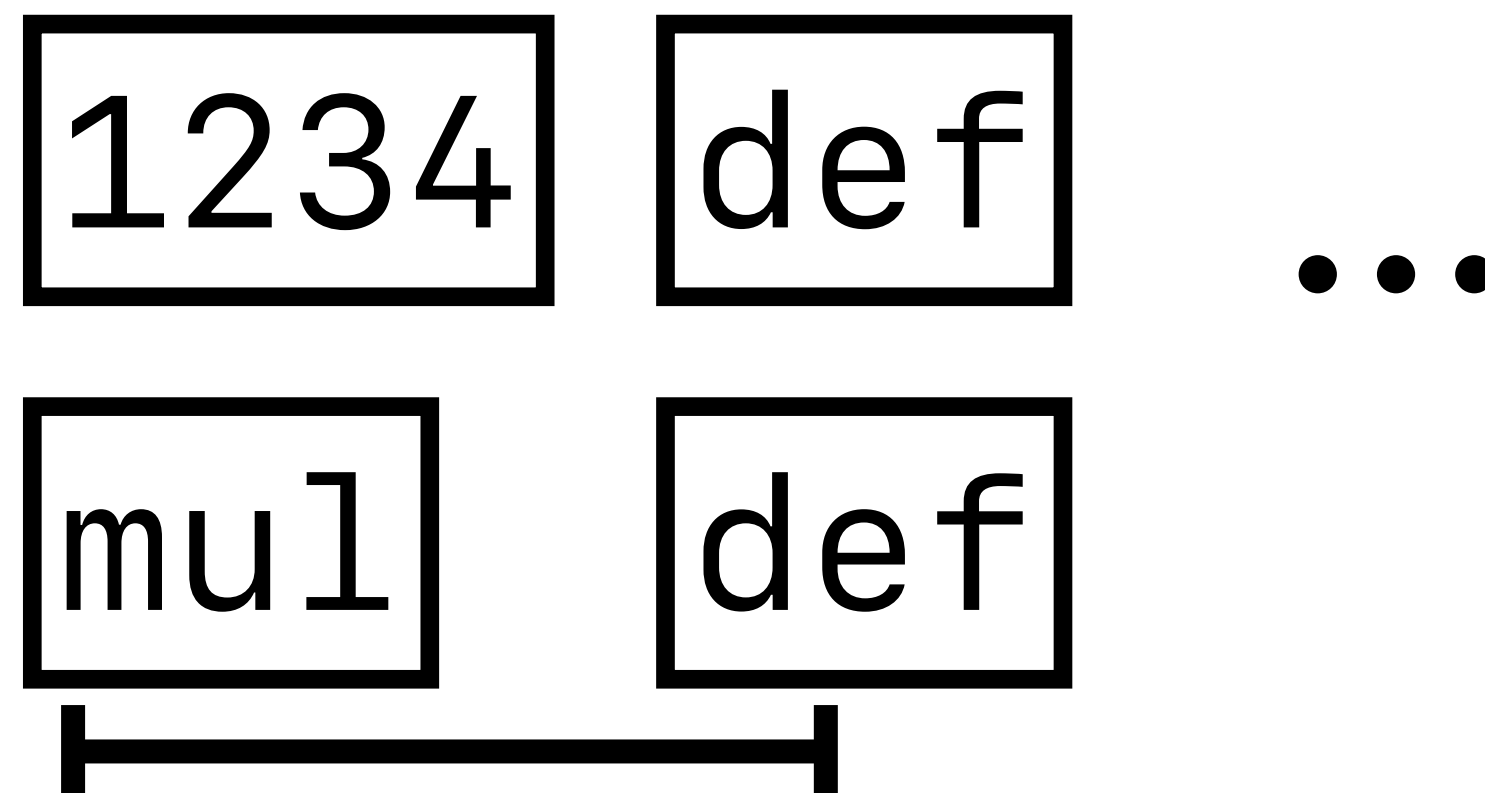
Identifier rule matches strings starting with muld

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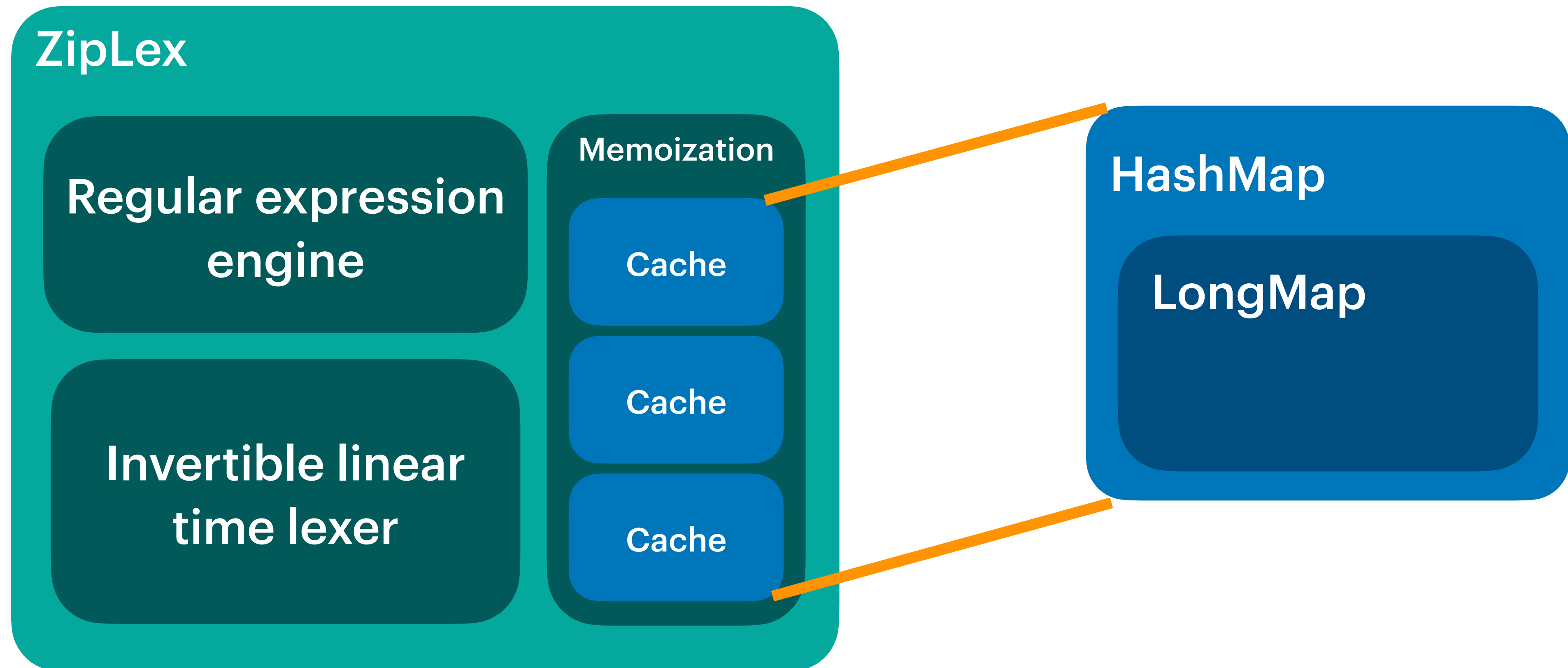
1234 def ...



When 2 tokens are separable, the longest prefix by any rule is always the first token

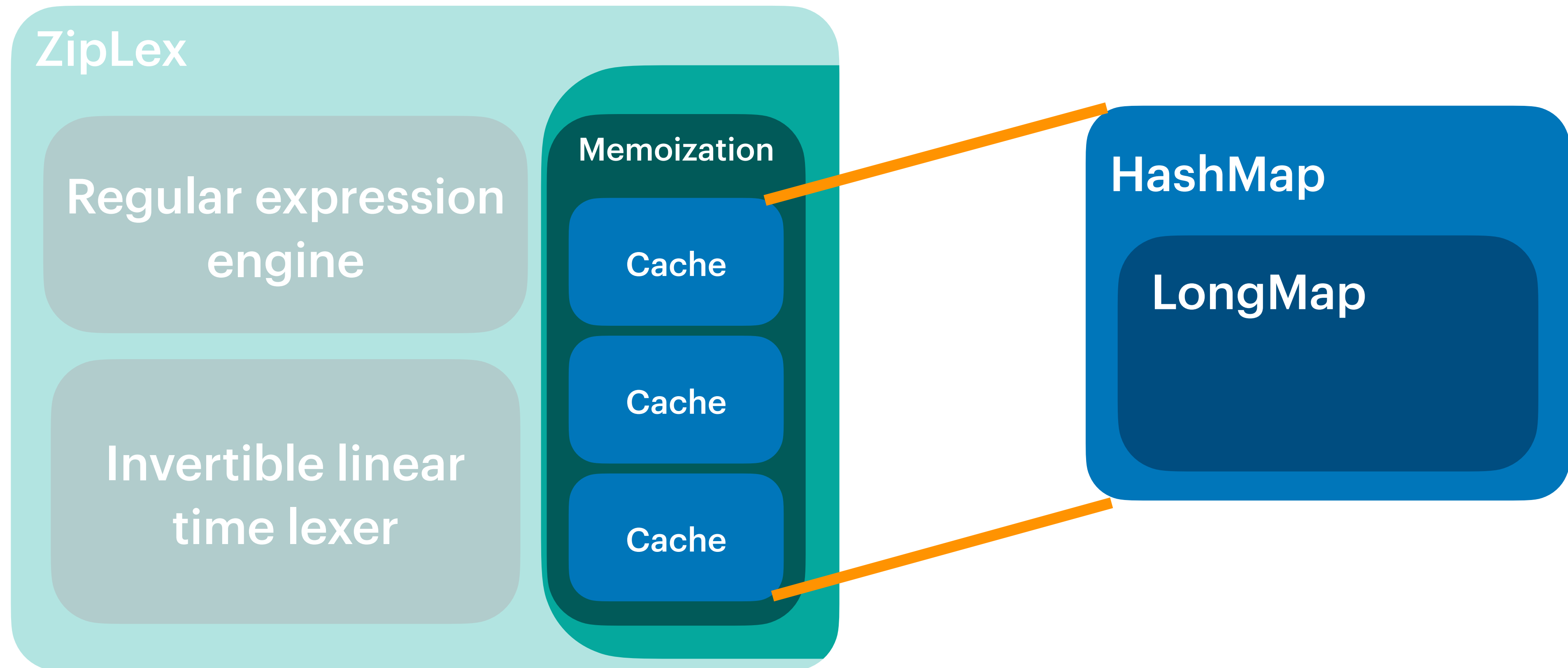
ZipLex Framework

Overview



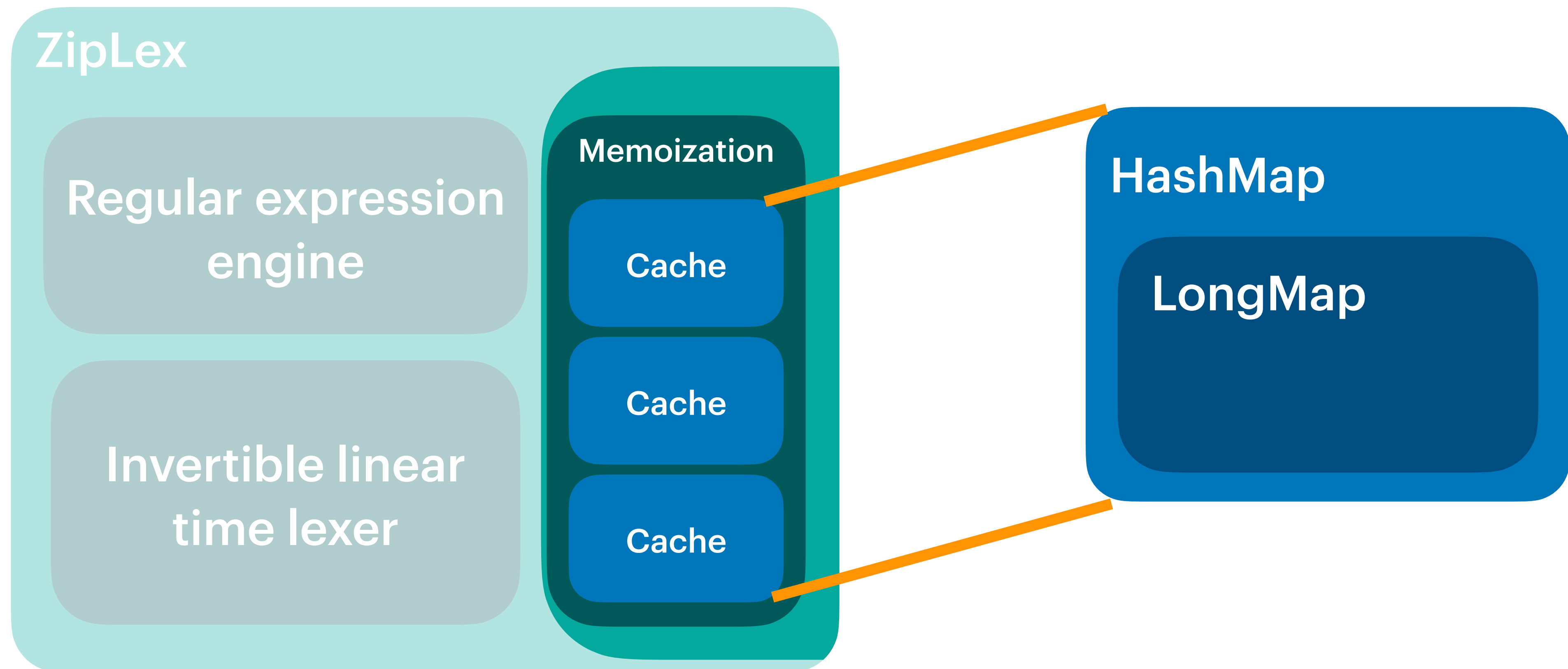
ZipLex Framework

Overview



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Reusable Verified Memoization Framework

Implementation

Reuse LongMap¹

LongMap[List[(K, V)]]

¹**Chassot, S.**, Kunčák, V. (2024). **Verifying a Realistic Mutable Hash Table**. In: IJCAR 2024. Lecture Notes in Computer Science, vol 14739. Springer

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- **Verified mutable** Hash Table

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Regular expression matching and lexing

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Memoized zipper-**derivatives computation** yielding **DFAs**

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Lexing

Memoization enables **linear complexity** lexing (Reps 1998)¹

¹Reps, Thomas. 1998. “‘Maximal-Munch’ Tokenization in Linear Time.” *ACM Trans. Program. Lang. Syst.* 20 (2): 259–73. <https://doi.org/10.1145/276393.276394>.

Memoization

Regular expression matching and lexing

Regular expression engine

Memoized zipper-**derivatives computation** yielding **DFAs**

Lexing

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Longest prefix computation

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Project Scale

Scala project formally verified using Stainless

Module	Program	Spec + Proof	Total
Longest Match Lexing	580	2030	2610
Lexer Printing + Invertibility	130	1430	1560
Regex Derivative Based Matching	470	2440	2910
Zipper Based Matching	390	2670	3060
BalanceConc	255	320	575
List & Set lemmas	-	1610	1610
Mutable HashMap (excluding LongMap)	90	2700	2790
Total	1'915	13'200	15'115

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Mutable LongMap	410	8200	8610

Performance analysis

Performance

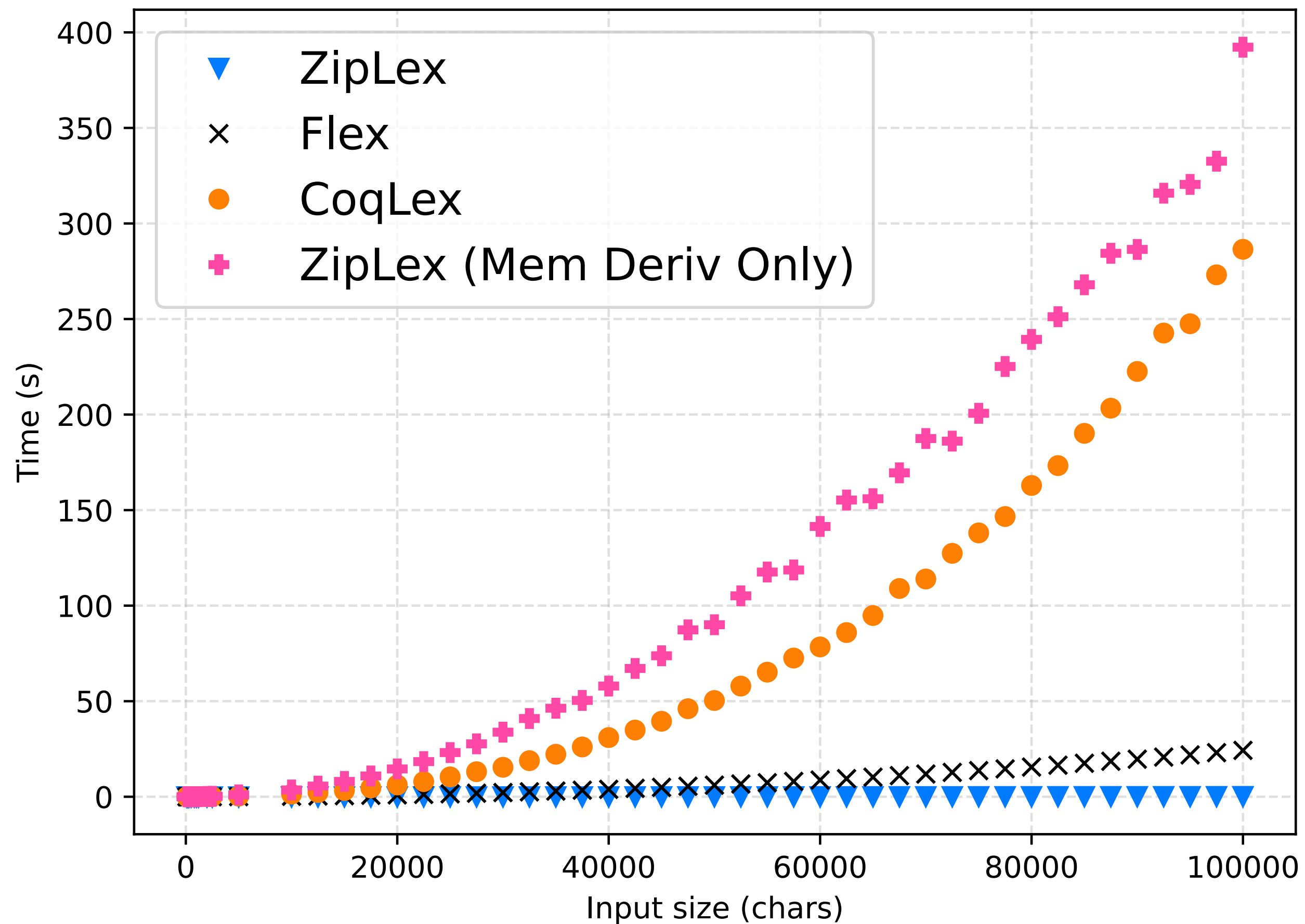
a and a^*b Benchmark

Input: " $aaa\dots a$ "

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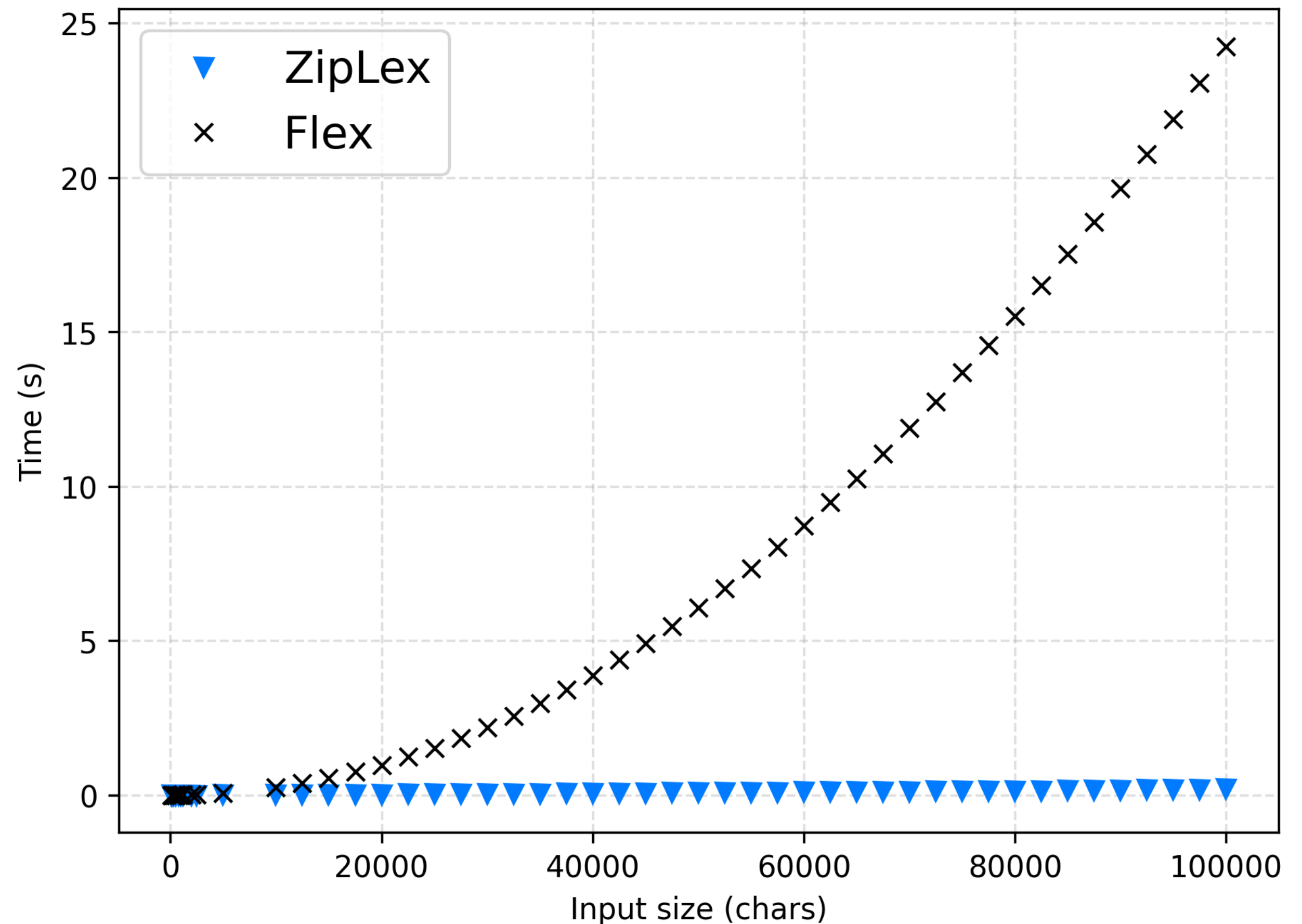
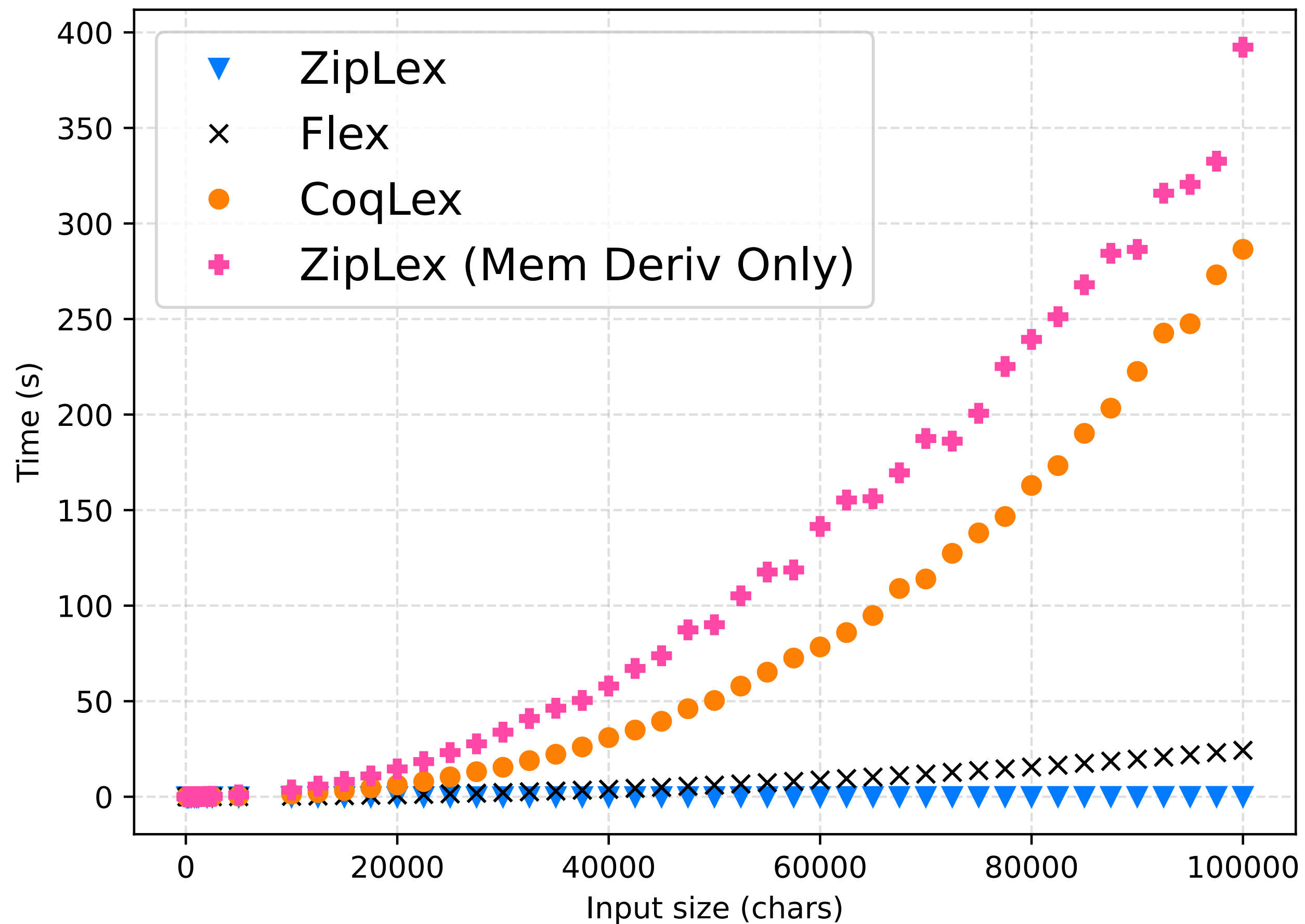
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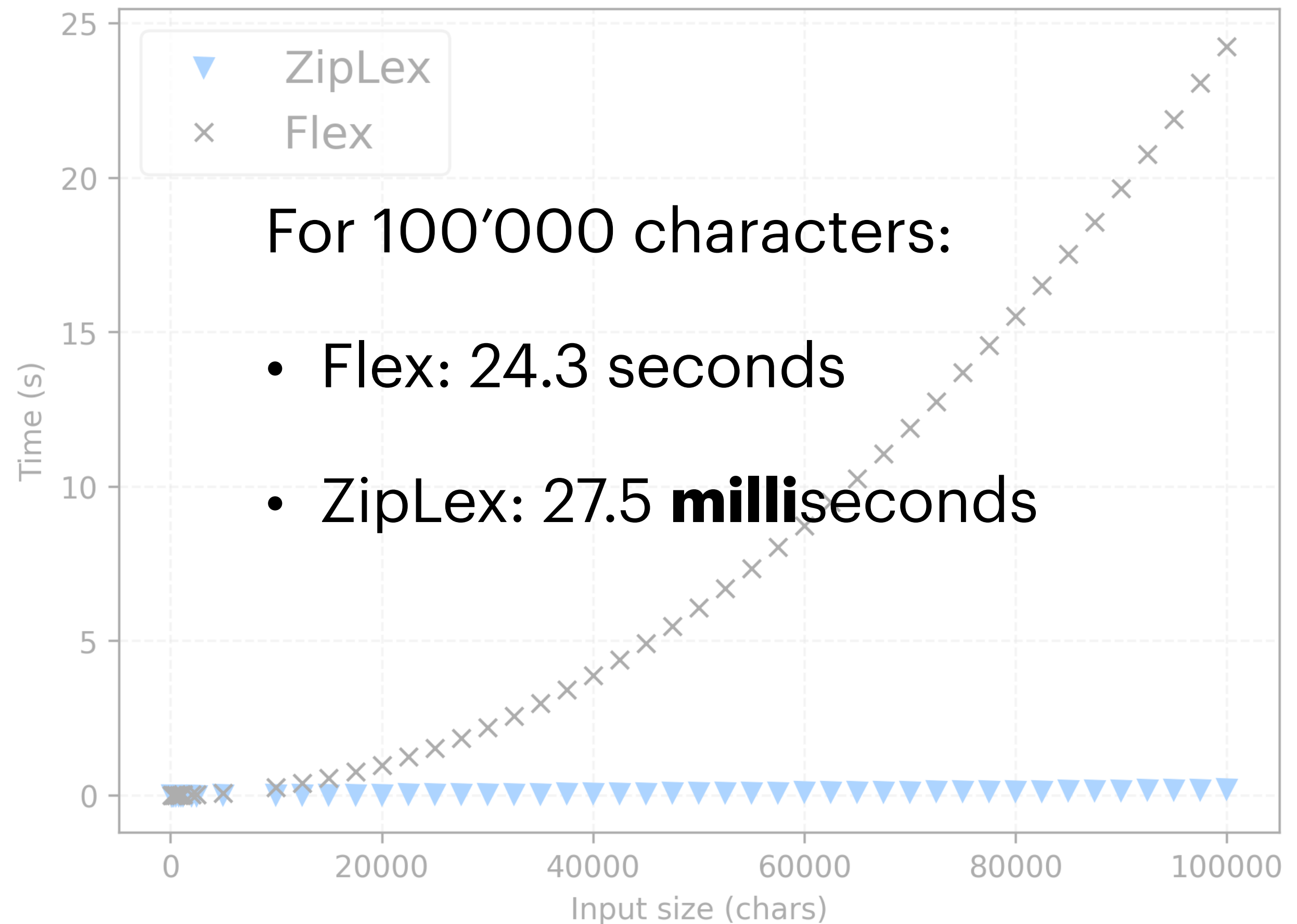
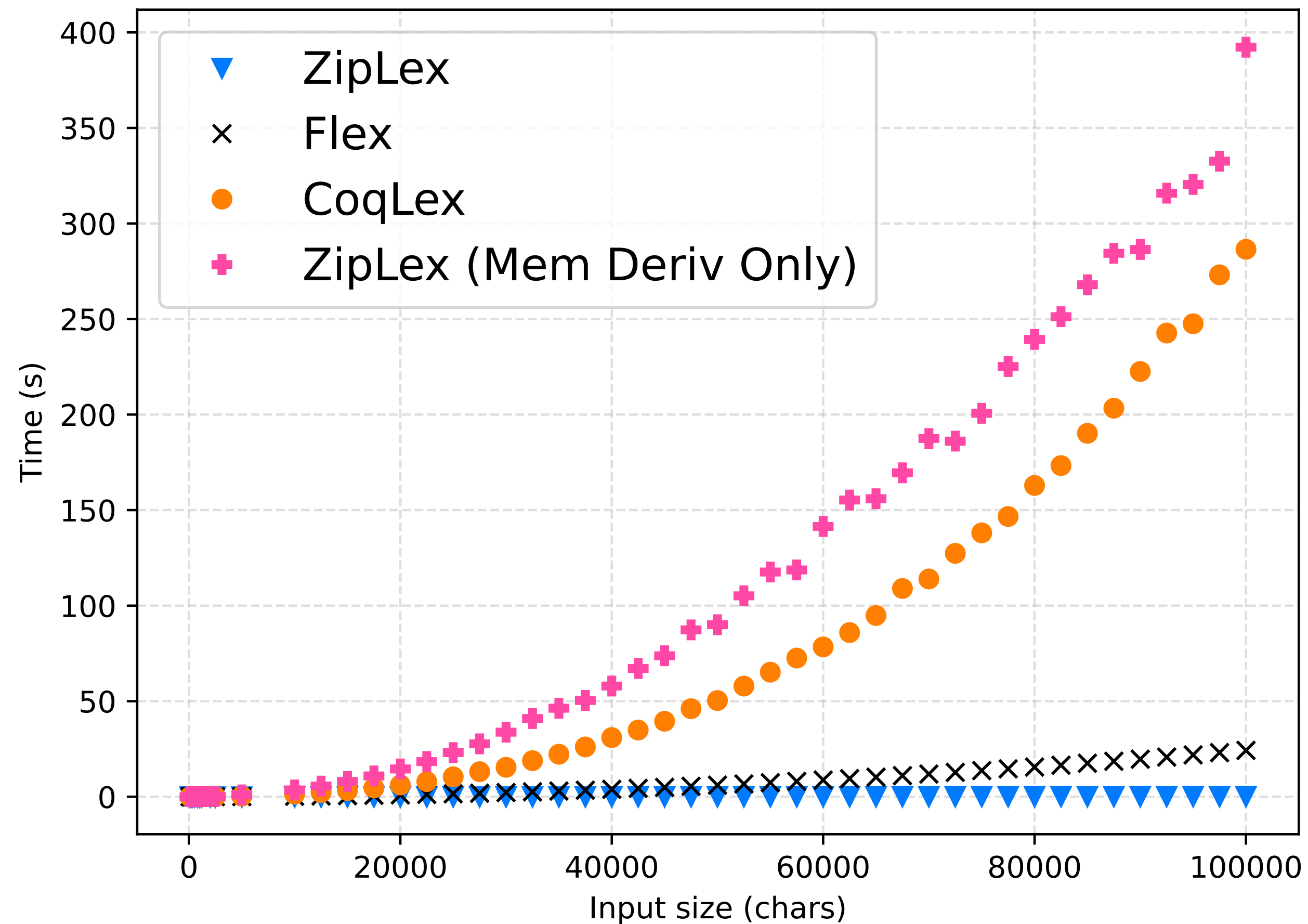
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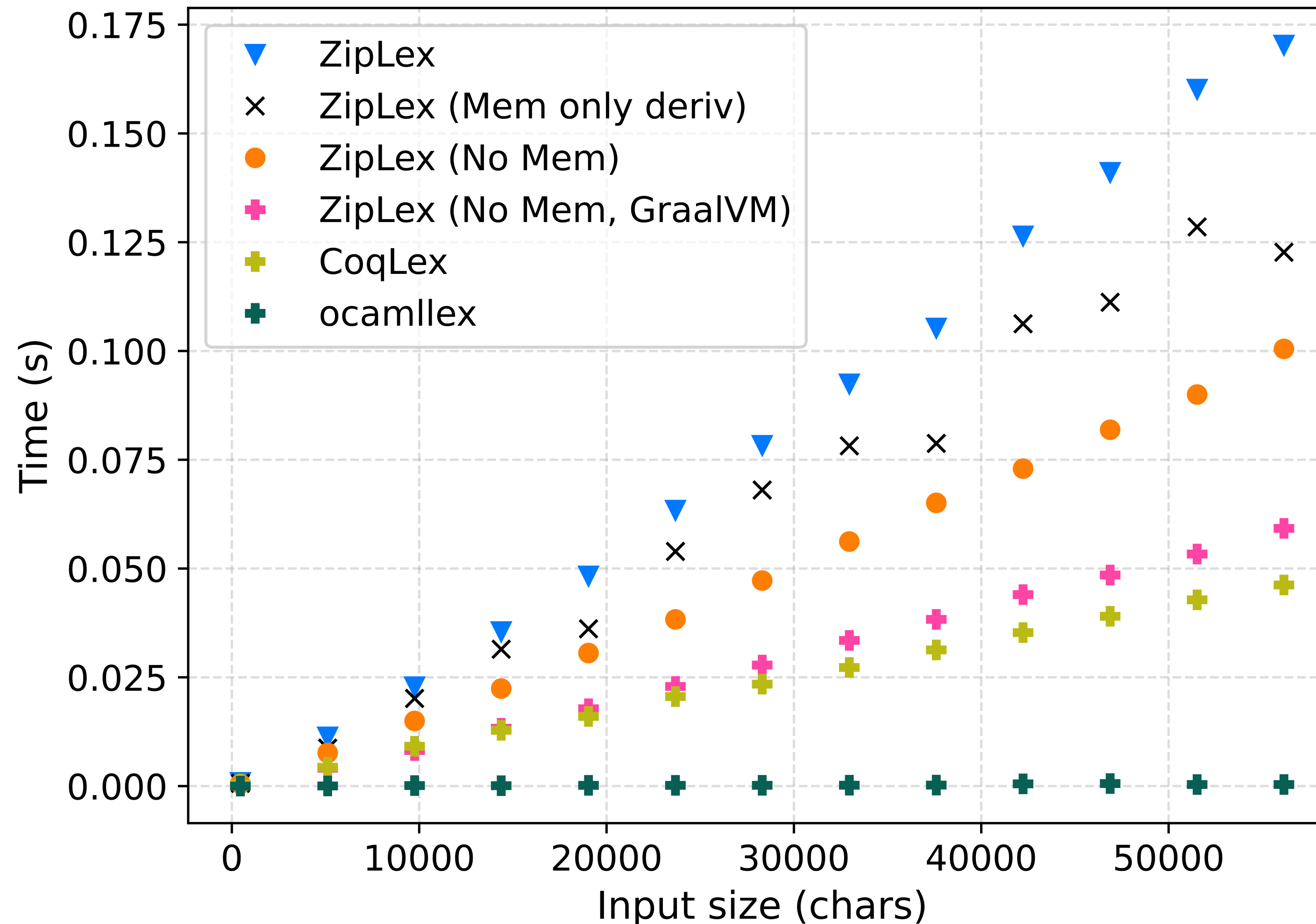
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Performance

JSON Benchmark



Input: JSON
objects and
arrays¹

Coqlex works
on ASCII
alphabet and
does not verify
invertibility

¹Ouedraogo, Wendlasida, Gabriel Scherer, and Lutz Strassburger. 2023. “Coqlex: Generating Formally Verified Lexers.” *The Art, Science, and Engineering of Programming* 8 (1): 3.

Conclusion

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Conclusion

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Fully verified and **linear-time** lexer

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Longest match and **invertible** along with **R-Path** predicates

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Regular expression engine, using derivatives with **zippers** and **memoization**

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[Stainless](#)



[ZipLex](#)

Backup

Token Separability

Examples

Token Separability

Examples

$$r_1 = a^+ \quad r_2 = b^+$$

Token Separability

Examples

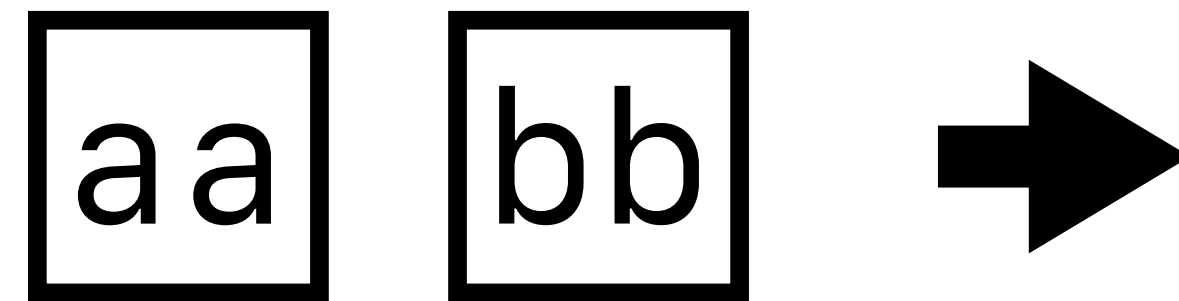
$$r_1 = a^+ \quad r_2 = b^+$$

aa bb

Token Separability

Examples

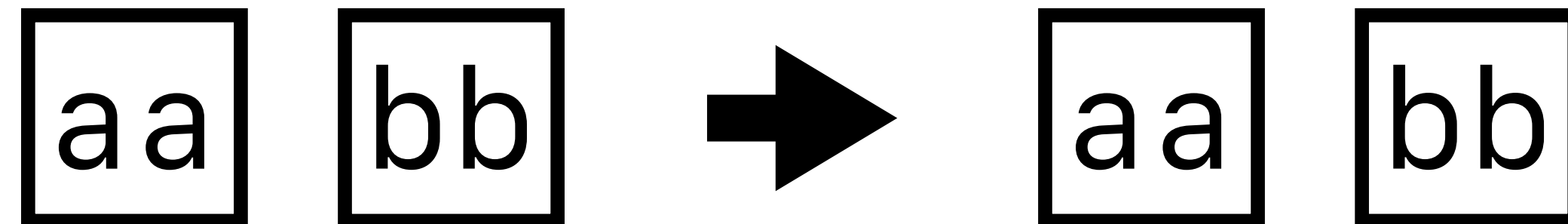
$$r_1 = a^+ \quad r_2 = b^+$$



Token Separability

Examples

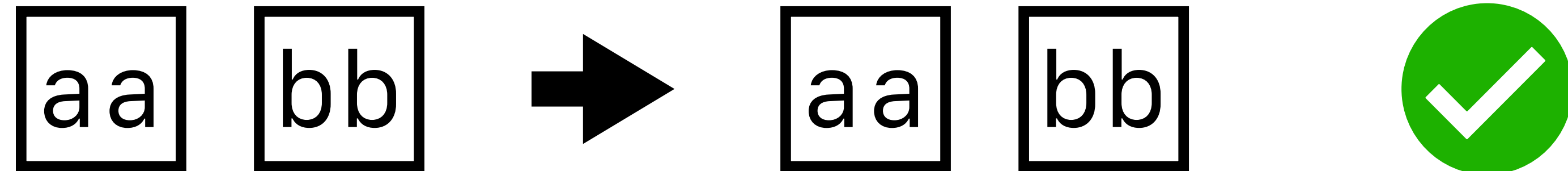
$$r_1 = a^+ \quad r_2 = b^+$$



Token Separability

Examples

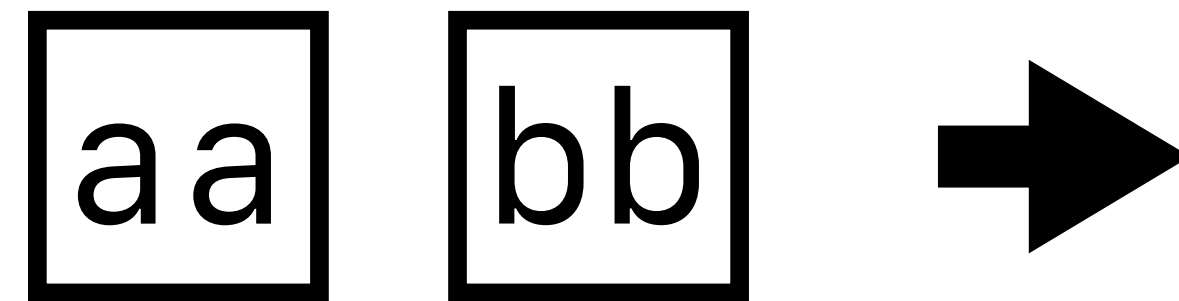
$$r_1 = a^+ \quad r_2 = b^+$$



Token Separability

Examples

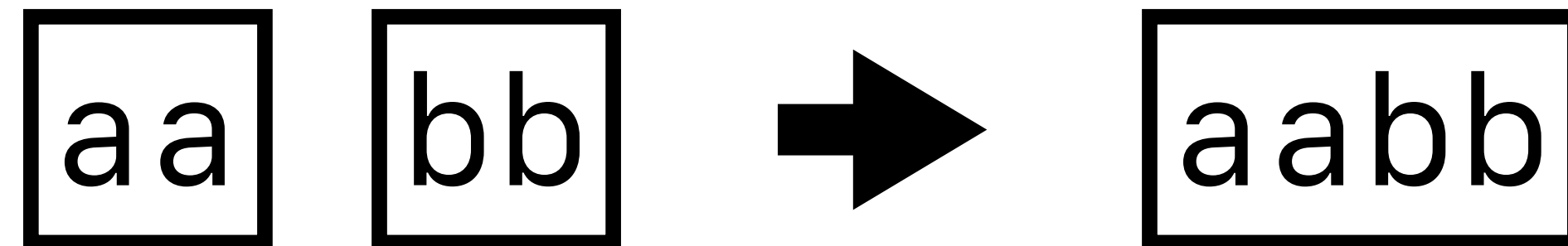
$$r_1 = a^+ \quad r_2 = b^+ \quad r_3 = (a|b)^+$$



Token Separability

Examples

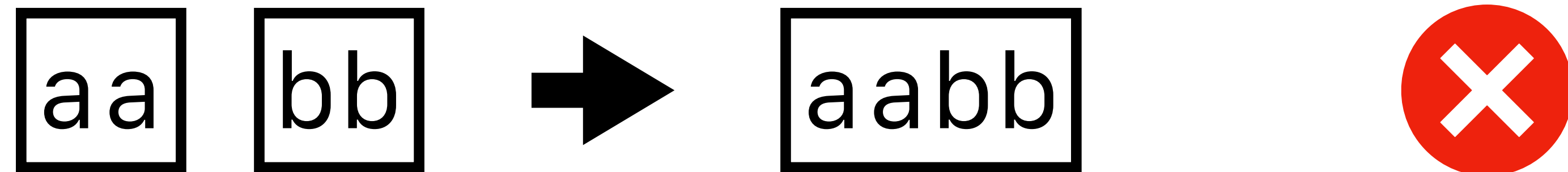
$$r_1 = a^+ \quad r_2 = b^+ \quad r_3 = (a|b)^+$$



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Examples

$$r_1 = a^+ \quad r_2 = b^+ \quad r_3 = (a|b)^+$$

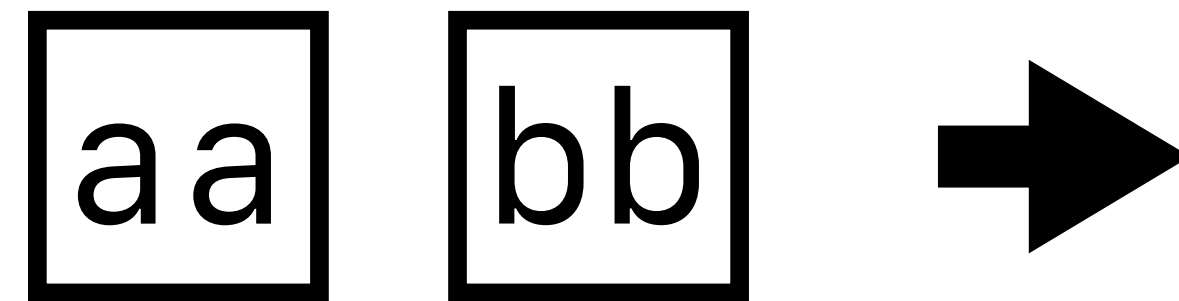


Separability is affected by ALL the rules

Token Separability

Examples

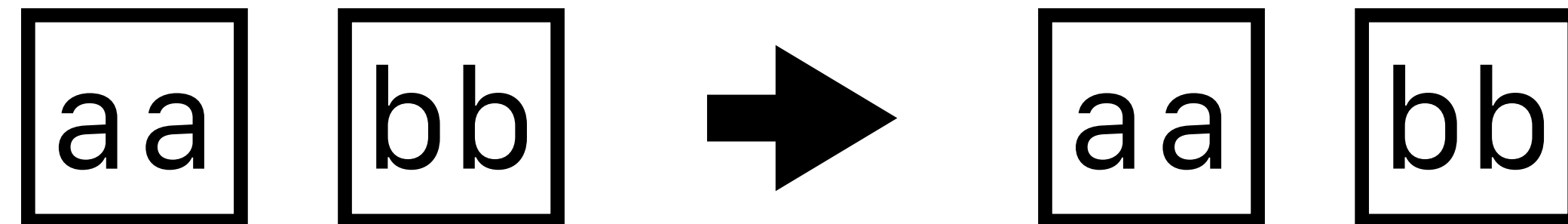
$$r_1 = a^+ \quad r_2 = b^+ \quad r_3 = (a \mid b)^+ \cdot c$$



Token Separability

Examples

$$r_1 = a^+ \quad r_2 = b^+ \quad r_3 = (a \mid b)^+ \cdot c$$



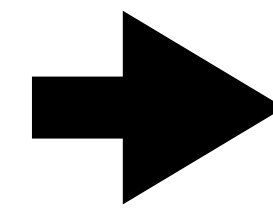
Token Separability

Examples

$$r_1 = a^+ \quad r_2 = b^+ \quad r_3 = (a \mid b)^+ \cdot c$$

aa

bb



aa

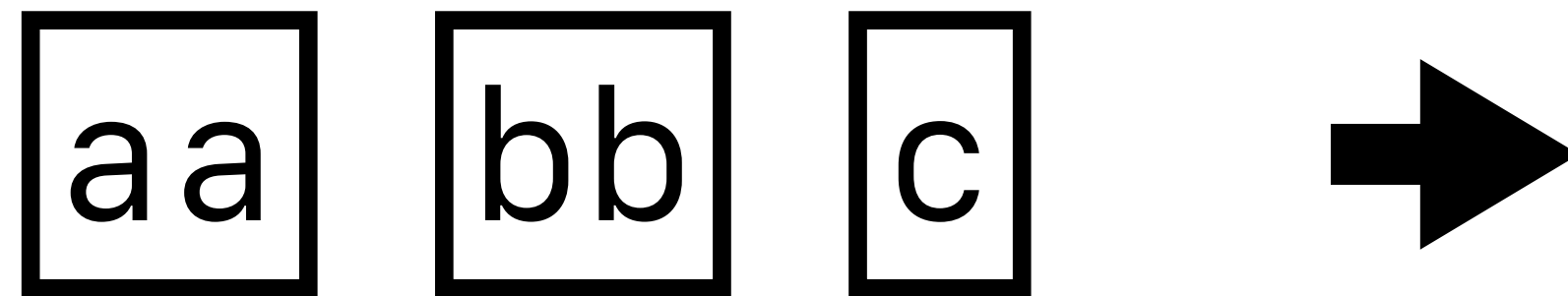
bb



Token Separability

Examples

$$r_1 = a^+ \quad r_2 = b^+ \quad r_3 = (a \mid b)^+ \cdot c$$



Token Separability

Examples

$$r_1 = a^+ \quad r_2 = b^+ \quad r_3 = (a \mid b)^+ \cdot c$$



Token Separability

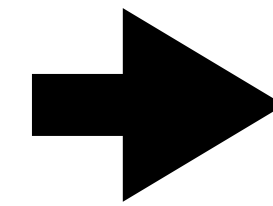
Examples

$$r_1 = a^+ \quad r_2 = b^+ \quad r_3 = (a \mid b)^+ \cdot c$$

aa

bb

c



aabbbc



Token Separability

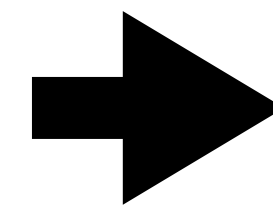
Examples

$$r_1 = a^+ \quad r_2 = b^+ \quad r_3 = (a \mid b)^+ \cdot c$$

aa

bb

c



aabbbc



Separability is affected by following tokens

Performance

a and a^*b Benchmark

Input: " $aaa...a$ "

Performance

a and a^*b Benchmark

Input: " $aaa\dots a$ "

